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**Numerical implementation of an interior point
method for linear optimazation**

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ملخص:

في البرمجة الخطية المعتمدة على طريقة النقاط الداخلية باستخدام دالة الحاجز اللوغاريتمية، تستخدم طرق التصنيف وولف والدالة التقريب الكبرى لاختيار طول الخطوة المناسب في كل تكرار وتهدف هذه الطرق إلى ضمان انخفاض دالة الهدف مع الحفاظ على بقاء الحلول داخل المجال المقبول، مما يحسن استقرار الخوارزمية وسرعة تقاربها وكفاءتها العددية.

الكلمات المفتاحية: البرمجة الخطية. طرق النقاط الداخلية. دالة الحاجز اللوغاريتمية. التصنيف. وولف. دالة التقريب الكبرى. التحليل العددي. تحسين عددي

Dans la programmation linéaire basée sur la méthode des points intérieurs avec la fonction barrière logarithmique, les méthodes de dichotomie, de Wolfe et de fonction majorante sont utilisées pour déterminer une longueur de pas appropriée à chaque itération. Leur objectif est d'assurer une diminution de la fonction objectif tout en maintenant les itérés à l'intérieur du domaine réalisable. Elles contribuent à améliorer la stabilité, la convergence et l'efficacité numérique de l'algorithme.

Mots clés : Programmation linéaire. Méthodes de points intérieurs. Barrière logarithmique. Dichotomie. Wolfe. Fonction Majorante. Analyse numérique. Optimisation convexe.

Summary :

In linear programming based on the interior-point method with a logarithmic barrier function, the dichotomy, Wolfe, and majorant function methods are used to determine an appropriate step size at each iteration. Their purpose is to ensure a decrease in the objective function while keeping the iterates within the feasible region. They improve the stability, convergence, and numerical efficiency of the optimization algorithm.

Key words: Linear programming. Interior point methods. Logarithmic barrier function. Dichotomy. Wolfe. Majorant approximation function. Numerical analysis. Convex optimization.