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Master's Thesis in Mathematics

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Theme :

**A variant of inverse barrier method with a relaxed
barrier term**

Presented by

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تتناول هذه الدراسة حل مسألة تحسين غير خطية باستعمال طريقة الحاجز العكسي، يحسب الاتجاه بطريقة نيوتن، بينما تحسب خطوة الانتقال بطريقة التراجع مع الاستقطاب التكميلي. يدعم هذا العمل بتجارب عددية مشجعة.

الكلمات المفتاحية:

مسائل الأمثلة غير الخطية، طريقة الحاجز العكسي، بطريقة التراجع، الاستقطاب التكميلي.

Abstract

This study concerned the resolution of nonlinear optimization problem using the inverse barrier technique. The search direction is calculated using Newton method, while the step size is determined by backtracking with cubic interpolation technique. This work is supported by encouraging numerical simulations.

Keywords : *Nonlinear Optimization, Inverse Barrier Method, Rebroussement Method, Cubic Interpolation.*

Résumé

Cette étude traite la résolution d'un problème d'optimisation non linéaire avec barrière inverse. La direction de recherche se calcule avec Newton, le pas de déplacement est déterminé par la technique de rebroussement avec interpolation cubique. Ce travail est soutenu par simulations numériques encourageantes.

Mots clés : *Optimisation non linéaire, Barrière inverse, Méthode de rebroussement, Interpolation cubique.*

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