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Chemotaxis and PDE's

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Abstract

This work is devoted to the rigorous mathematical analysis of a class of chemotaxis systems featuring a logistic source. Under appropriate initial-boundary conditions, we establish the global-in-time existence and uniform boundedness of solutions, successfully ruling out any finite-time blow-up. Furthermore, the asymptotic behavior is thoroughly investigated to accurately characterize the long-term dynamics of the system.

Keywords: Chemotaxis, Logistic source, Global existence, Boundedness, Asymptotic behavior.

Résumé

Ce travail est dédié à l'analyse mathématique rigoureuse d'une classe de systèmes de chimiotactisme dotés d'une source logistique. Sous des conditions initiales et aux limites appropriées, nous établissons l'existence globale en temps et la bornitude uniforme des solutions, excluant ainsi toute explosion en temps fini. De plus, le comportement asymptotique est minutieusement examiné afin de caractériser avec précision la dynamique à long terme du système.

Mots-clés: Chimiotactisme, Source logistique, Existence globale, Bornitude, Comportement asymptotique.

الملخص

يعنى هذا العمل بالتحليل الرياضي الدقيق لفئة من أنظمة الجذب الكيميائي المزودة بمصدر لوجيستي تحت شروط بدائية وحدية ملائمة تثبت الوجود العالمي عبر الزمن والمحدودية المنتظمة للحلول، مما ينفي تماماً أي احتمال للانفجار في زمن منته علاوة على ذلك، تم استقصاء السلوك المقارب بعمق لتوصيف الديناميكية طويلة المدى للنظام بدقة.

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