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Solving a Fuzzy Capacitated Two-Index Transportation Problem

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تُعَدُّ مسألة النقل نموذجاً أساسياً في بحوث العمليات. تهدف إلى تقليل تكاليف التوزيع مع احترام قيود العرض والطلب. تستند الصياغات الكلاسيكية لهذه المسألة إلى افتراض أن جميع معاملات النموذج معروفة بدقة، وهو مالا يتحقق في التطبيقات الواقعية إلا نادراً. ففي الممارسة العملية، غالباً ما تتسم تكاليف النقل الوحدوية، أو الكميات المتاحة، أو الطلبات بعدم اليقين أو الضبابية. ولأخذ هذه الجوانب بعين الاعتبار، تم إدخال مفاهيم المنطق الضبابي، والمجموعات الضبابية، والأعداد الضبابية. إن هذه الأدوات تتيح صياغة وتحليل مسألة النقل ثنائية المؤشر مع القدرات في بيئة ضبابية. يُسهم دمج النظرية الضبابية في نماذج النقل في تمثيل أكثر واقعية لحالات اتخاذ القرار، وبشكل أداة فعالة لدعم القرار. في هذه المذكرة، نركز على دراسة وحل مسألة النقل ثنائية المؤشر مع القدرات في سياق ضبابي. ولتقييم أداء طريقة الحل المقترحة، أجرينا دراسة عددية مقارنة باستخدام حلاً برمجية خطية متطور كمرجع. ننوه أن النتائج التي تم الحصول عليها واعدة وتؤكد فعالية الطريقة.

كلمات دلالية: البرمجة الخطية، مسألة النقل الكلاسيكية، مسألة النقل ذات المؤشرين مع السعات، خوارزمية النقل، الرياضيات الضبابية.

Abstract

The transportation problem is a fundamental model in operations research. It aims to minimize distribution costs while satisfying supply and demand constraints. The classical formulations of this problem rely on the assumption that all model parameters are perfectly known, which is rarely the case in real-world applications. In practice, unit transportation costs, availabilities, or demands are often subject to uncertainty or imprecision. To account for these aspects, the concepts of fuzzy logic, fuzzy sets, and fuzzy numbers have been introduced. These tools make it possible to formulate and analyze the two-index capacitated transportation problem in a fuzzy environment. The integration of fuzzy theory into transportation models provides a more realistic representation of decision-making situations and constitutes a particularly effective decision-support tool. In this dissertation, we focus on the study and resolution of a two-index capacitated transportation problem in a fuzzy context. To assess the performance of the proposed approach, we conducted a comparative numerical study with an advanced linear programming solver used as a benchmark. The obtained results are promising and confirm the effectiveness of our approach.

Keywords: Linear programming, classical transportation problem, Capacitated two-index transportation problem, Transportation Algorithm. Fuzzy mathematics.

Résumé

Le problème de transport occupe une place centrale en recherche opérationnelle. Il consiste à déterminer un plan de distribution minimisant le coût global de transport, sous des contraintes d'offre et de demande. Dans sa formulation classique, ce problème suppose que l'ensemble des paramètres – coûts unitaires, quantités disponibles et demandes – est connu avec certitude. Toutefois, cette hypothèse est rarement satisfaite dans les contextes réels, où les données sont souvent entachées d'incertitude ou d'imprécision. Afin de pallier cette limite, les concepts de logique floue, d'ensembles flous et de nombres flous ont été introduits. Ils offrent un cadre théorique adapté à la modélisation des données imprécises et permettent d'étudier le problème de transport à deux indices avec capacités en environnement flou. L'apport de la théorie floue réside dans sa capacité à mieux représenter la complexité des situations réelles et à fournir une aide à la décision plus robuste. Le présent mémoire est consacré à l'étude et à la résolution de ce problème en contexte flou. Afin de valider l'approche proposée, nous avons procédé à une comparaison numérique avec un solveur d'optimisation linéaire performant, utilisé comme référence. Les résultats numériques obtenus sont satisfaisants et attestent de la pertinence de notre démarche.

Mots-clés : Programmation linéaire, Problème de transport classique, Problème de transport à deux indices avec capacités, Algorithme de transport, Mathématiques floues.