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Par

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THÈME

Recherche des coefficients de déport optimaux de correction des engrenages cylindriques en développante de cercle

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Devant le jury composé de:

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THESIS

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By

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SUBJECT

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Symbol	Description	Unit
a	Center distance (mm)	mm
a'	Working center distance (mm)	mm
α_n	Normal pressure angle	(°)
α_t'	Pressure angle at the pitch cylinder	(°)
b	Face width	mm
β	Helix angle	(°)
d	Reference diamter	mm
d_a	Tip diamter	mm
d_b	Base diamter	mm
ε_α	Transverse contact ratio	—
ε_β	Overlap ratio	—
g_α	Length of path contact	mm
g_f	Approach path length	mm
g_a	Recess path length	mm
γ	Specific sliding coefficient	—
K_A	Application factor	—
K_v	Dynamic factor	—
$K_{H\beta}$	Face load distribution factor	—
$K_{H\alpha}$	Transverse load distribution factor	—
$K_{F\alpha}$	Transverse load factor (root stress)	—
$K_{F\beta}$	Face load factor (root stress)	—
m_n	Normal module	mm
m_t	Transverse module	mm
n_1	Rotational speed	rpm
P	Transmitted power	kW
ρ_f	Tooth root radius at the critical section	mm
s_{at}	Arc thickness of the tooth at the tip diameter	mm
$S_{H\,\min}$	Minimum safety factor for surface durability	—
$S_{\mathrm{F\,min}}$	Safety factor for tooth breakage	—
$\sigma_{F\,\lim}$	Nominal stress number (bending)	N/mm²
$\sigma_{H\,\lim}$	Allowable stress number (contact)	N/mm²
u	Transmission ratio	—
v_g	Sliding velocity	m/s

v_r	Rolling velocitys	m/s
$x_{1,2}$	Profile shift coefficient of pinion (or wheel)	—
Y_{NT}	Life factor for tooth root stress	—
Y_β	Helix angle factor (tooth root)	—
Y_ε	Contact ratio factor (tooth root)	—
Y_{Fa}	Tooth form factor	—
Y_{Sa}	Stress correction factor	—
$Y_{\delta relT}$	Relative notch sensitivity factor	—
Y_{RrelT}	Surface factor	—
Y_x	Size factor (tooth root)	—
Y_{ST}	Stress correction factor	—
Z_ε	Contact ratio factor (pitting)	—
Z_β	Helix angle factor (pitting)	—
Z_{NT}	Life factor for contact stress	—
Z_v	Velocity factor	—
Z_E	Elasticity factor	$(N/mm^2)^{1/2}$
Z_H	Zone factor	—
Z_L	Lubricant factor	—
Z_R	Roughness factor affecting surface durability	—
Z_x	Size factor (pitting)	—
Z_W	Work hardening factor	—
z_n	Virtual number of teeth of a helical gear	—
$z_{1,2}$	Teeth number of pinion (or wheel)	—
$\omega_{1,2}$	Angular velocity of pinion (or wheel)	rad/s
$g_i(x)$	Inequality constraints design	—
$h_j(x)$	Equality constraints design	—
$f(x)$	Objective function	—
β_i	Weighting factors	—
λ_i	Penalty factors	—

CHAPTER ONE:

Introduction and background

INTRODUCTION AND BACKGROUND

1.1. Introduction

The search for the best compromise between economic, mechanical and technological imperatives has always been the primary aim of the mechanical engineer. Most design optimization problems in structural engineering are highly non-linear, include several constraints on stresses, displacements, load carrying capability, and geometrical configuration (Gandomi et al., 2011). In addition, they involve different mixed design variables (integer, discrete and continuous). Discrete variables are employed in numerous manners like the representation of the set of standard-sized components, the decision on the number of identical parts or the choice between different design options (Cao et al., 2000). Examples of such variables are: standard wire diameter of spring, thicknesses of sheets, number of gear teeth, value gear module, selection choice of ball bearings, etc.

Gears are the most important components used as a part of mechanical systems to transmit motion and power between rotating shafts by means of progressive engagement of projections called teeth. They are widely used in industry for advantages of compact structure, large power transmission and high efficiency. Gears have been used since ancient times, according to Dowson (1979) around 300 B.C. in Alexandria wooden gears were used for toys, catapults, etc. Today gears have been involved in almost every type of machinery. Gear transmission systems are widely used in a variety of industry applications, such as automotive, wind turbines, mining, airplanes, helicopters and marine vessels. The cylindrical gears are the simplest and the most popular type used in the mechanical machinery. They connect and transmit power/rotation between parallel shafts, economically they are the least expensive to manufacture.

However, the gear design is a complicated task involving multiple objectives which often conflict such as strength, pitting resistance, bending stress and scoring wear. As well, involves many empirical formulas, different graphs, charts and tables, which lead to a complicated design. The complex shape and geometry of a gear leads to a large number of mixed design parameters (integer: number of teeth, discrete: normal module, and real: gear width). There are several highly non-linear constraints on the main operating performances of the gear such as tooth bending strength, tooth surface durability, tooth surface fatigue, efficiency, contact ratio, thickness of the tooth tip diameter, interferences and other geometric conditions.

1.2. A review of gear design optimization

In the last decades many researchers have paid attention on the problem of gear optimization design by using different methods such as: computer program, conventional optimization techniques, meta-heuristic algorithms, as well as developed software and optimization tools. Though many studies have been conducted on gear design optimization, only important and relevant literatures connected to the present work are discussed. In the following, a state of the art of gear design optimization is collected and presented.

1.2.1. Gear design optimization: Using computer programs

The optimization of gears using the aid of computers has been a subject of many research reports. In fact, Cockerham and Waite (1976) in their paper developed a computer aided procedure for designing spur and helical gears of standard form with 20 degree pressure angle and involute profile based on strength and wear considerations. The developed program was fully integrated, requiring only design specifications and material properties to be supplied. They used the British standard BS 436: 1940 for the strength and wear calculations. Through this approach, the authors concluded that for the complex gear trains of more than two gears it is a relatively simple task to calculate the torque and speed requirements manually for use in the program.

Further, Tong and Walton (1987) described an interactive program named Internal Gear Design (IGD) for designing internal spur and helical gear pairs. In this paper, the program first performs a kinematic analysis to determine tooth numbers and to satisfy centre distance requirements, and then proceeds to calculate the face width. In addition, the program allows the user to experiment with different solutions and so a form of optimization can be used. For the stresses calculations the British standard BS 436: 1940 was used. In the same way, Madhusudan and Vijayasimha (1987) in their work presented a computer program which was capable of designing a required type of gear under a specific working condition. The authors' work was oriented towards the training and education aspects of the gear design process.

Similarly, Li et al. (1996) developed a method for the minimization of centre distance of helical gears using computer by exploiting inherent relations in gear design. They established the upper and lower boundaries for the search interval of centre distance by determining the limiting values of American Gear Manufacturers Association (AGMA) geometry factors. In this paper, the program is tested for various gear design requirements. In addition numerous real engineering design problems were solved successfully. The authors concluded that the transformation of geometry constraints results in a fairly small feasible domain for the combination of module and numbers of teeth, and making it possible to find the maximum power capacity combination by enumeration without high computational cost.

With the aid of computer, the gear design can be carried out iteratively and the decision variables which satisfy the given conditions can be determined. However, the decision variables obtained maybe satisfying certain conditions or just only one condition at a time. For instance, if the module is calculated based on bending strength, the same module is substituted to calculate the surface durability. It is accepted if the strength of surface durability is within the limit, otherwise it is changed accordingly. Consequently the obtained design does not guarantee the optimum one (Savsani et al. 2010).

1.2.2. Gear design optimization: using classical optimization techniques

Besides, the increasing demand for high strength, efficient, quiet and high precision gears forces the designer to use the optimal design methodology. In the literature several researches were reported for the optimization of gears using different optimization techniques. For example, Osyczka (1978) in his paper formulated a problem for finding the basic constructional parameters of a gearbox to minimize simultaneously four objective functions, which are volume of elements, peripheral velocity between gears, width of gearbox and distance between axes of input and output shafts. The author used numerical methods considering min-max principle of optimality along with Monte Carlo method and trade-off studies. A detailed example considering a lathe gearbox optimization problem was presented.

Savage et al. (1982) optimized the design of a standard gear mesh with the objective of minimizing the centre distance for a given gear ratio, pinion torque and the allowable tooth strength. The developed procedure applied to both internal and external gearing. Furthermore, a number of kinematic and gear strength constraints were considered, and a design procedure established to find the optimal design. A design space is defined in terms of the number of teeth on the pinion and the diametric pitch. According to Tong and Walton (1987) the authors ignored gear ratio tolerances and did not allow for any practical variations on face width limits. Helical gear sets and the problem of profile shift modifications were ignored also. In order to find the optimal design a graphical method was used by the authors. Moreover the efficiency of the proposed method was illustrated through a practical example.

Further, Carroll and Johnson (1984) presented an optimal design technique to obtain compact standard spur gear, where the design objective was to minimize the center distance or the pinion diameter. The paper expanded the work of Savage et al. (1982) by explicit consideration of the AGMA dynamic and geometry factors. A FORTRAN program was used in this work and the developed code was applicable to spur gear with standard tooth system only. To solve the optimization problem gradient based algorithm was used. The authors conclude that the developed approach can be extended for helical and bevel gears.

Optimization of double speed reduction gear trains has been attempted by Prayoonrat and Walton (1988), where BS 436: 1940 was used in order to determine the satisfactory tooth strengths. The algorithm was based on a two stage optimization process, the first employed a mathematical optimization method and the second used the direct search method and heuristic iteration approach. The objective of the optimization was the minimum centre distance between the input/output shafts and the lays haft. The algorithm was successfully used to optimize and design the gear train for a real gearbox to be used for winding wire ropes. The final design showed that the overall dimension was reduced by some 100 mm over existing designs.

Zarefar and Muthukrishnan (1993) in their work used a modified adaptive random search algorithm to minimize the weight of helical gears. They performed the module, the helix angle, the number of teeth on pinion, and the face width as the design variables. The contact and the bending stresses were modeled as constraints for the problem. Further, the proposed methodology allows for the implementation of non-linear design functions and constraints without the need for linearization. Also, the technique developed by the authors has the capability of starting from either feasible regions or infeasible regions of the design domain. Moreover, the tested examples for the helical gears demonstrated the feasibility of the proposed technique as a practical method for design optimization and the viability is further illustrated in a performance test against the conjugate gradient search method.

Wang and Wang (1994) used a Modified Iterative Weighted Tchebycheff (MIWT). Four objectives have been considered by the authors including the gear size, weight of meshing gear set, tooth deflection and life of gear sets. Moreover, the results were compared with those from previous research. In another work, Savage et al. (1994) presented a new method for optimum design of compact spur gear reductions. This approach included the design of one spur gear pair, two shafts and four bearings. The authors used the polynomial interpolation to deal with the discrete bearing properties and proportions into continuous variables, then after finding the continuous optimum, the designer can analyze near optimal designs for comparison and selection. The purpose of the design optimization was the total weight of the speed reducer and bearings life duration. In addition, the inequality constraints restrict the designs to have adequate tooth bending strength, tooth scoring resistance, involute contact geometry, shaft strength and stiffness at the full AGMA estimated dynamic load. The optimization procedure was carried out by using the modified feasible directions search algorithm. The found results exposed the optimal configurations of the bearings.

In Chang et al. (1997) a mathematical model of a modified helical gear train, manufactured with a practical hobbing machine has been developed. The model takes into account the center-distance variation and axial misalignment. In addition, the tooth contact analysis and kinematic errors of a gear due to mis-assembly are investigated. A multiple optimization method was applied to reduce the level of gear kinematic errors, and to investigate optimal gear tooth modifications. In this study, two numerical examples were presented to illustrate the kinematic optimization of the proposed helical gear train.

Thompson et al. (2002) presented a generalized optimal design formulation with multiple objectives which is applicable to a gear train of arbitrary complexity. The authors applied the methodology to the design of two-stage and three-stage spur gear reduction units, subject to identical loading conditions and design criteria. The developed approach serves to extend traditional design procedures by demonstrating the tradeoff between surface fatigue life and minimum volume using a basic multi-objective optimization procedure. The optimization problem was solved by employing quasi-Newton method. At conclusion, the authors recommended that a potential extensions of the work would include incorporation of more detailed (e.g. AGMA) design standards and extension to larger-scale, generalized problems which could include helical gearing, multiple planetary gear sets, and other more complex systems.

Designing gear pairs are complex and it requires the use of complex nonlinear constraints, discrete design variables, as well as non-differentiable functions. Nevertheless, most traditional optimization methods are appropriate for simple objective functions and applied to problems where gradient information is available. Moreover, such techniques are not robust and often fail to solve optimization problems that have many local optima (Rao and Savsani, 2012; Saravanan, 2006). Consequently, the classical methods are no more effective in solving the problems of gears design optimization.

1.2.3. Gear design optimization: using meta-heuristic algorithms

In order to overcome the drawbacks of traditional optimization techniques, various meta-heuristic algorithms are developed and used in order to optimize the design of different gears. It is worth mentioning that the meta-heuristic approaches can efficiently deal with this kind of problems quickly and with higher precision. In Boris et al. (1996) an expert system for designing and manufacturing a gearbox is developed. The optimum dimensions of gears are determined based on a two stage optimization process. In the first the Genetic Algorithm (GA) was used to determine the optimal dimensions of a gearbox. In the second stage the expert system took technological requirements into account including the selection of cutting tool and cutting conditions, the special sequence of machining, and the tolerances. The strength verification process was based on DIN 3990 calculation and other methods like the finite element method and boundary element method. The performance of the system was evaluated through two study examples. According to the authors the results showed a reduction in gearing masses by approximation 32 % which gives a reduction in price of approximation 40 %.

By using improved GA, Yokota et al. (1998) solved an optimal weight design problem of a spur gear. In this work, the authors formulated the problem of gear with constraints of bending strength of gear, torsional strength of shafts and gear dimensions. However, certain constraints were not satisfied and the obtained solution was not optimum. In the same way, Marcelin (2001) in his paper applied the GA to minimize the volume of a single stage speed reducer. In order to treat the constraints functions the penalty approach has been used. The author concluded through the results found, for the case study problem, that GA is a very efficient method to optimize gears.

Deb and Jain (2003) used a Non-dominated Sorting GA (NSGA-II) in order to solve a multi-objective optimization of a multi-speed gearbox. The proposed algorithm in this work is capable of solve the problem with mixed discrete and real valued parameters and is also capable of finding multiple non-dominated solutions in a single simulation run. The optimization problem involves the maximization of the power and minimization volume of the gear box. A total of the twenty-nine mixed type of decision variables (integer, discrete, and continuous), five equality and ninety-six nonlinear inequality constraints are included. Marcelin (2004) employed the GA for multi-criteria design optimization of the gears including minimum gears volume, minimum center distance as well as maximum efficiency.

Li et al. (2008) introduced an optimal design problem of a two-stage speed reducer. The purpose of the study was to obtain the multi-objective optimization design scheme of a gear reducer. An adaptive GA based on a Fuzzy controller (FGA) was used. The authors employed the fuzzy technique to adjust the weights of objective functions, crossover probability, mutation probability, crossover positions and mutation positions. The optimization problem formulated for nine mixed design variables. The numerical results found for the test problem showed the advantages of the applied approach and the effectiveness of an adaptive GA.

Gologlu and Zeyveli (2009) in their study automated the preliminary design of a gearbox with spur, helical and bevel gears using the GA algorithm. Gear volume was formulated as objective function along with contact stress, bending stress, number of teeth on gear and pinion, module and face width of gear as constraints. The design problem involves seven mixed variables. Furthermore, both static and dynamic penalty functions were presented to handle the design constraints. The results obtained by GA were compared with a deterministic design procedure, and showed a 98 % of the success.

Mendi et al. (2010) employed also the GA approach to minimize the total volume of a gearbox. The design problem involves nine mixed variables. Additionally, the penalty approach was used to treat the constraints. In order to facilitate the evaluation of the GA a program has been developed by the authors based on Borland Delphi 6.0 platform. The results obtained by GA optimization were compared to those obtained by analytical method. The comparison indicates that the volume obtained by GA is 1.47 % lower than analytical method.

Buiga and Popa (2012) in their paper presented an optimal design mass minimization problem of a complete single-stage helical gear unit using GA. Padmanabhan et al. (2013) used the GA to optimize a simple planetary gear. They investigated the minimizing of the volume of two stage gear trains with mixed variables. The optimization problem formulation has been based on the recommendation of AGMA standard to satisfy the required design specification of gear trains. Buiga and Tudose (2014) optimized the mass of a two stage helical coaxial speed reducer by using the GA. The objective function was described by a set of seventeen mixed design variables and seventy six constraints. The results demonstrated that the method presented by the authors, offers better design solutions compared to traditional design method.

Sanghvi et al. (2014) optimized the volume and the load carrying capacity of two-stage helical gear. To solve the problem the authors used three different methodologies including MATLAB optimization toolbox, GA and multi objective optimization (NSGAII). In the first two methods, volume is minimized in the first step and then the load carrying capacities of both shafts are calculated. In the third method, the problem is treated as a multi objective problem. For the optimization purpose, face width, module, and number of teeth are taken as design variables. Constraints are imposed on bending strength, surface fatigue strength, and interference. The authors concluded that NSGA-II shows more superior results than obtained by other methods in terms of both objectives. The volume decreased by the 13.08% and load carrying capacity increased by 1%.

Further, Thoan et al. (2015) presented the optimization design of stress relieving holes at root fillet in spur gear. The design method consists of two parts, in the first part, a finite element model that is combined with two gear segments for pinion gear and driven gear is built and the bending stress is analyzed. In the second part, the sizes and the locations of the stress-relieving holes of the gear teeth are optimized in order to reduce the root fillet stress. The authors used the GA methodology to find out the optimized locations and sizes of the holes at the root fillet. An illustrated example shows that the maximum principal stress in gear root fillet has been reduced by 14.69 % based on the proposed optimization design.

According to the above literature, it can be said that the GA algorithm maybe is the most commonly non-traditional optimization algorithm used to optimize the gears design. However, as reported by

Sabarinath et al. (2013) the GA algorithm provides a near optimal solution for a complex problem having large number of variables and constraints. This is mainly due to the difficulty in determining the optimum controlling parameters such as population size, crossover rate and mutation rate.

Therefore, several researchers solved the problem of gears design by using other meta-heuristic algorithms, which are powerful, robust and able to provide an accurate solution. In Chong et al. (2002) a new methodology to design multi-stage gear drives has been presented. The algorithm consists of four steps to automate the preliminary design. In the final step, the positions of the gears were determined to minimize the geometrical volume of a gearbox by using Simulated Annealing (SA) algorithm. The effectiveness of the methodology validated by design examples of four-stage gear drives and generated considerably good design results in both aspects of the dimensional and the configuration design.

Zhao et al. (2007) developed in their work a multi objective Ant Colony System (ACS) to provide a solution for the reliability optimization problems of series-parallel system and also demonstrated its application to the reliability design of gearbox. According the authors, the multi objective ACS algorithm offered distinct advantages to these problems compared to alternative optimization methods, and can be applied to a more diverse problem domain with respect to the type or size of the problems.

Savsani et al. (2010) used two advanced optimization algorithms including Particle Swarm Optimization (PSO) and SA to find the optimal combination of design parameters for the minimum weight of a spur gear train. They considered the same optimization problem formulated previously by Yokota et al. (1998). The design problem was modified according to AGMA equations which contain six design variables and eight constraints. The results obtained by using PSO and SA algorithms showed significant improvement over the results obtained in (Yokota et al., 1998) using a GA. Wang and Zhen-yu (2011) used also the SA algorithm to find the optimal combination of design parameters for minimum weight of gears. For the application on an example a simple and multi-stage spur gear trains were considered.

In Gang et al. (2011) the design optimization of two-stage cylindrical helical gear reducer is discussed. The volume and the center gear distance are adopted as objective functions separately, involving discrete mixed design variables. In order to solve the optimization problem the PSO algorithm was used by the authors. In addition, to adjust the convergence speed the inertia weight coefficient has been used in the PSO. The optimization result showed that the method developed by the authors is better than other optimization methods.

Guyonneau et al. (2013) used Monte Carlo method for optimizing geometry for the flank of a gear-tooth by minimizing the equivalent contact stress. The stress calculation method is based on Hertz theory. The geometric variation of the flank of the tooth is achieved relative to the involute profile. During this optimization, a polynomial expression of the tooth geometry is used. The Monte Carlo simulation is compared with analytical propagation.

Tamboli et al. (2014) in their paper presented the study on the optimal design of heavy duty helical gear pair using PSO technique. The objective function was the minimization of volume of cylindrical gear pair. The face width, the gear teeth, the helix angle and the module were chosen as the design variables. Various factors of size and strength of gears are computed for gear geometry parameter using DIN 3990: 1987 and DIN 3960: 1987 standards.

Padmanabhan et al. (2015) proposed a new Modified Artificial Immune (MAIS) algorithm to optimize spur gear design. They considered the maximum power, the minimum volume and the maximum gear efficiency as the objectives of the gear design. In order to generate a superior population of initial solutions, a two new cloning operator has been introduced. Then, two phased mutation process adapted to make a good enhancement in potential solution. The optimized results obtained by the proposed algorithm showed considerable reduction in weight in compared with trail method for both gear materials and also shows better results over power and center distance.

Recently, Salomon et al. (2015) formulated an optimization problem of a gearbox for a random variant of torque and speed requirements. Both the gear numbers and their characteristics were optimized to minimize the overall energy consumption and the gearbox cost. The authors employed an Active Robustness (AR) methodology to solve the problem. The results show that this approach can find a set of robust designs, revealing a trade-off between energy efficiency and production cost. This can serve as a useful decision-making tool for the gearbox design process, as well as for other applications.

Panda et al. (2016) optimized the weight of a spur gear using a novel DE algorithm using a novel DE algorithm. The weight of a spur gear was formulated as a constrained optimization problem using the same formulation reported in (Savsani et al. 2010). In addition, a total of six design variables corresponding to gear geometry and material property have been considered. The obtained results have been compared with that of GA, SA, and PSO. The authors reported that the optimization results founded are encouraging in terms of objective function values and speed convergence time. Further, using the optimum geometric variables a CAD model has been prepared and imported into the ANSYS workbench for stress analysis.

For the same problem, Rao (2016) made an effort to verify if any improvement in the solution is possible by employing two new algorithms Teaching Learning Based Optimization (TLBO) and Elitist Teaching Learning Based Optimization (ETLBO). The results of the TLBO and ETLBO algorithms are compared with the GA, SA, and PSO algorithms. The computational results showed that the TLBO and ETLBO algorithms have obtained more accurate solutions than those obtained by using the other optimization methods. Zolfaghari et al. (2017) in their study utilized SA and GA coupled with AGMA instructions to design the straight bevel gears pair. The optimization purpose was the gear volume minimization, using the teeth number, module and face width as design variables. The authors tested the capability of the method through a design example.

Recently, Fu et al. (2018) applied a new meta-heuristic algorithm named Crow Search Algorithm (CSA) to optimize the design of hoisting transmission system from heavy cable shovel excavator. Three objectives have been considered, including the minimum volume of the system, maximum surface fatigue life and maximum load capacity. To illustrate the effectiveness of the algorithm, the authors compared the optimal results provided by CSA with other methods.

1.2.4. Gear design optimization: using software and optimization tools

In addition to the previously mentioned methods, there have been a number of studies uses the software and toolbox in order to optimize gears. Indeed, Huang et al. (2005) developed an interactive physical programming approach in order to optimize a three-stage spur gear reduction unit. The objectives have been the minimum volume, the maximum surface fatigue life, and the maximum load-carrying capacity The MATLAB constrained optimization package is used to solve this problem. An optimization problem of three-stage spur gear reduction unit was given to illustrate the effectiveness of the proposed approach.

Wei et al. (2009) developed a mathematical model of optimization considering the basic design parameters, mainly tooth number, modulus, face width, and helix angle of gearbox as design variables and reduction of weight or volume as an objective. The model is illustrated by an example of the gearbox of medium-sized motor truck. Optimization tool box of MATLAB and sequential quadratic programming (SQP) method were used to optimize the gearbox. The design criterion and performance conditions of gearbox are treated as constraints. Padmanabhan et al. (2010) have made attempt to optimize the design of spur gear pair by using the GA approach and an analytical tool named MITcalc. Moreover, the Finite Element Analysis (FEA) was carried out and results were compared with the allowable limit.

Zhang et al. (2010) realized an automatic optimization design for single spur gear reducer, based on using visual basic programming mixed with MATLAB. The mathematical model was developed with an aim to maximize the working life of bearings and minimize the size of the reducer. Moreover, Long et al. (2011) used MATLAB Toolbox to make the size of the reducer smallest and minimize the requirement of materials. In this work the model of two-stage helical gear reducer was created and then objective function, design variables and constraint condition was described. The obtained results for the case study showed a reduction in gearing masses by a 35.28 % and center distance reduced by 33 mm.

Jhalani and Chaudhary (2012) discussed the various parameters which can affect the design of the gearbox for knee mounted energy harvester device and later it frames the optimization problem of mass function based on the dimensions of gearbox for the problem. The authors solved the problem using multi-start approach of MATLAB global optimization toolbox and value of global optimum function is obtained considering all the local optimum solutions of problem.

Further, Marjanovic et al. (2012) in their study provided a practical approach for gearbox optimization with spur gears based on an optimal selection of materials, position of shaft axes and gear ratio. This approach it was the basis for the development of gear train optimization (GTO) software. Their results show that the volume of spur gear train is reduced by 22.5 %. Also, the developed software can be used for problems with Multi-objective optimization.

Wan and Zhang (2012) in their study formulated an optimal design problem of a spur gear drive with a fixed load factor. Three methods were presented to find the globally optimal design scheme on the structure of the spur gear pair. In the first method, by suitable variable transformation, the constructed model was converted into a linear program with mixed variables. By the second method, a global optimal design problem of spur gear drive with soft tooth flank has been solved in the continuous variable space if the modification of contact ratio factor is taken into account. The case studies showed the validity of the constructed model and the solution methods.

Golabi et al. (2014) minimized the volume of gearbox by using MATLAB optimization toolbox, where two and three stage gear trains have been considered. By choosing different values for the input power, gear ratio and hardness of gears the practical graphs from the results of the optimization has been presented. Recently, Wang et al. (2017) used also MATLAB optimization toolbox in order to minimize the volume of a spur gear. In this study, the model of clearance volume caused by bottom clearance and the modification coefficient has been established to estimate an exact total volume of the spur gear. An optimization example of a single-stage spur gear system has been illustrated.

1.3. Objectives and contributions

The main target of the present thesis is the optimization design of the involute cylindrical gears geometry by using an evolutionary algorithm. Among the existing approaches, the DE algorithm is chosen for its simplicity, its power and in particular the ease of implementation. In the same way, this work deals with the deterministic solving of single or multi-objective constrained engineering design optimization problems of mixed discrete continuous types. To perform these tasks, two new versions of DE algorithm are proposed. The developed methods aim also to treat the mixed variables, to avoid the manual choice of control parameter settings and to overcome the disadvantage of the slow convergence of the standard version of DE.

The first proposed algorithm, named Adaptive Mixed Differential Evolution (AMDE). In order to avoid the manual tuning of the mutation and crossover values the AMDE algorithm is based on self adaptive mechanism jDE proposed by Brest et al. (2006). Furthermore, a modified exterior penalty is applied to handle the design constraints. The second variant can be viewed as an improved version of AMDE and the algorithm is called modified AMDE. This variant adopted the superiority of feasible points to compare between the target vector and the trial vector in the selection step. In addition, the normalization of constraint violations to treat each constraint equally is performed for this variant. The performance of the developed algorithms are numerically tested using two sets of test problems including six widely used constrained engineering benchmarks and six real constrained mechanical design problems.

Moreover, we applied the proposed methods to solve the problem of the optimal geometry of the cylindrical gears. In first application, the optimal selection values of profile shift coefficients for cylindrical involute spur and helical gears is performed, using the DE/rand2/bin version. The optimization procedure is developed specifically for exact balancing specific sliding coefficients at extremes of contact path and account for gear design constraints. In the second step the AMDE and the modified AMDE algorithms are applied to optimize the profile tooth of a real spur gear in a large transport machine, where the objectives are to equalize the maximum bending stresses and the specific sliding coefficients at extremes of contact path. The mathematical model of the maximum bending stresses is developed using a FEA calculation. In a third step, the developed algorithms are applied in order to solve the problem of designing a single stage cylindrical speed reducer. The mathematical formulation of the problem takes into account all the standards recommendations specified for complete gear design. For the case study, two speed reducers are considered. Moreover, the optimal results of the developed algorithms are compared with those provided by traditional design method.

We should mention that the developing of CAD model and calculation by using FEA for optimization verification has been done by Dr. Ivana Atanasovska from the Mathematical Institute of the Serbian Academy of Sciences and Arts.

1.4. Thesis organization

This thesis consists of six chapters, and its structure can be summarized as follows. **Chapter 2** provides a fundamental review of cylindrical spur and helical gears with the involute profile. The chapter begins with the basic classification of the gears, and the nomenclature geometry of the cylindrical. In addition, the geometry of corrected gearing is discussed, then a review of profile shift coefficient selection is presented.

Chapter 3 introduces the concepts and definitions related to optimization problems. Principles classification of optimization methods is discussed. Then, the most techniques used to handle with constraints are presented and explained. The remainder of the chapter is reserved for a detailed overview on the differential evolution algorithm.

Chapter 4 presents the two variants of the DE algorithm developed in this thesis. Moreover, the performances of the proposed algorithms are numerically tested using two sets of test problems including six widely used constrained engineering benchmark and six real constrained mechanical design problems.

Chapter 5 presents, in the first part, the application of the developed algorithms in addition to DE/rand2/bin version to solve the problem of the optimal geometry of cylindrical gears. In its second part, the AMDE and modified AMDE are applied to solve the problem of designing a single stage cylindrical speed reducer. Finally, the chapter 6 draws conclusions from the results obtained through the present thesis, and also makes some suggestions for future works.

CHAPTER TWO:

Fundamental principles of cylindrical gear design

2.1. Introduction

Gear is the most efficient mechanical system used to transmit rotary motion and power from one shaft to another by means of progressive engagement of projections called teeth. It is widely used in mechanical devices and machines such as automobile, machine tools, wind turbines, gas turbine, marine and industrial power transmission. Among the several existing types of gears, the cylindrical ones are the simplest and the most used in the mechanical machinery, economically they are the least expensive to manufacture.

The present chapter provides information on basic gear classification in the second section. The geometry of tooth flanks of external cylindrical spur and helical gears with the involute profile is discussed in third section. The geometry of corrected gearing is presented in section four. Then a detailed review of profile shift coefficient selection is presented in section five. Finally, the last section is dedicated to gear manufacturing.

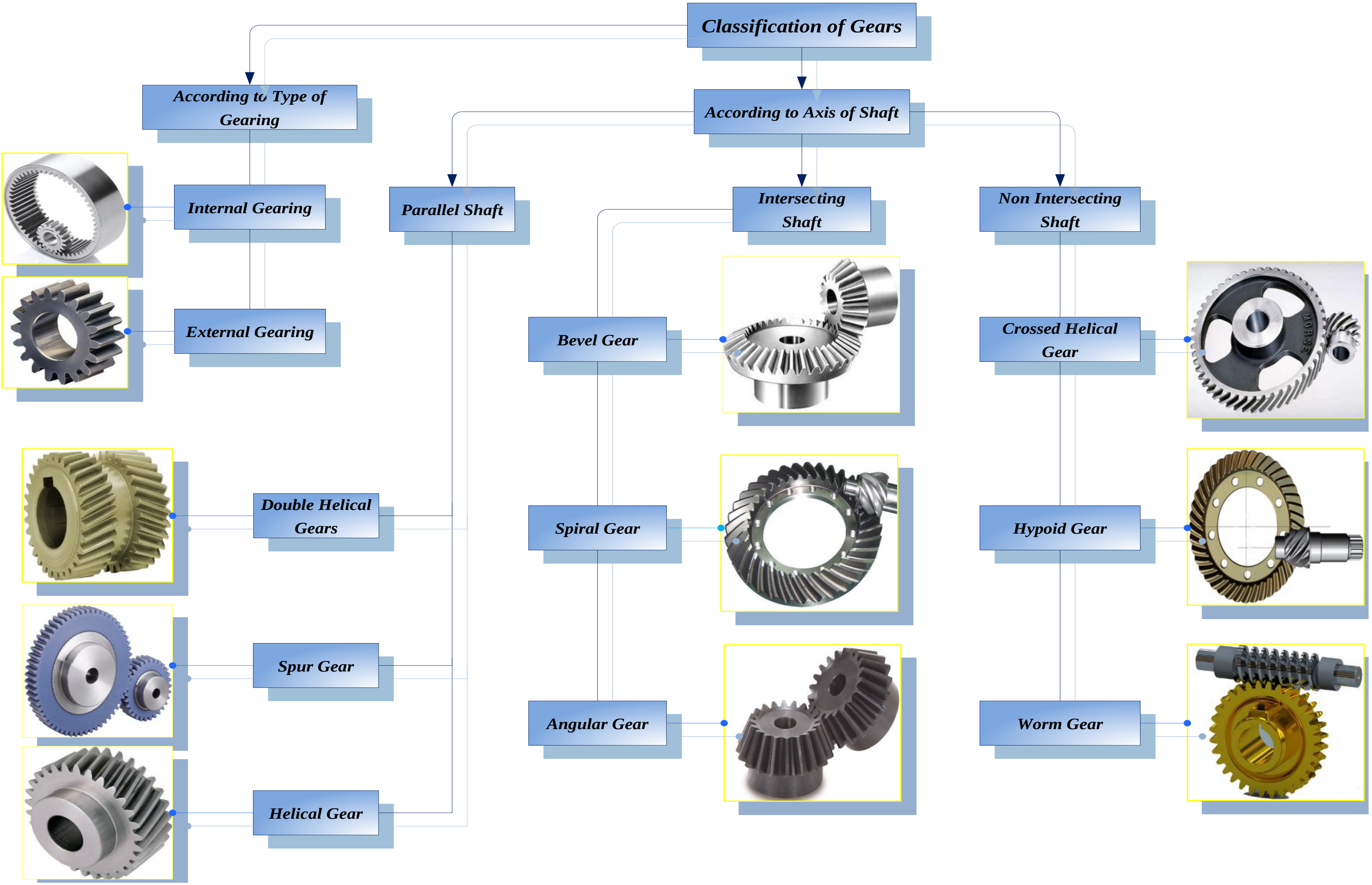
2.2. Basic types of gears

According to ISO 6336-1: 2006 gears operate in pairs the smaller being called «pinion» and the larger «wheel». Usually the pinion drives the wheel, and the system acts as a speed reducer and a torque converter (Maitra, 1994). There are different types of gears currently being designed, manufactured and used in the world. They can be classified into many types based on several criteria, the Figure 2.1 gives a flowchart of general gear classification.

Usually the gears are classified according to two main categories, depending on positions of the axes of revolution and the type of gearing. From the first point of view two kinds can be distinguished (Dudley, 1962), gears with parallel axes and gears with non parallel axes (intersecting and non-intersecting). Another classification relates to type of gearing, and the gear can be external or internal. For more details, there is an abundance of literature on the types of gears, their main characteristics, materials used and suitability of applications; we refer the interested readers to the following books (Henriot, 2007; Maitra, 1994; Stephen, 2010; Stephen, 2012; Stephen, 2016).

It may be mentioned that the present thesis deals only with the external cylindrical spur and helical gears with the involute profile.

Fig. 2.1 Gears classification



2.3. Cylindrical gears: geometry and nomenclature

The cylindrical gears are the simplest and the most popular type used in the mechanical machinery. They connect and transmit power/rotation between parallel shafts, economically they are the least expensive to manufacture.

In order to show the advantages of using this category of gearing in power transmission Heinz and Ralf (2016) presented in their book a detailed comparison between the cylindrical gears and other known type of gears (as shown in the Table 2.1) in terms of the range of gear ratios and the performance characteristics such as the transmission capacity the maximum rotational speed, the linear velocity and the efficiency of the gear.

Table 2.1 Threshold values achieved of listed gears (Heinz and Ralf, 2016)

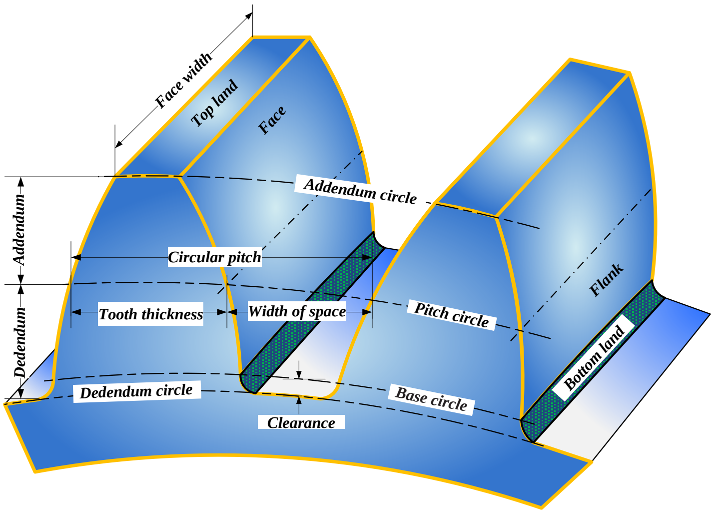
Gear	Power P in (kW)		Max rotational speed in (min^{-1})	Velocity v in (m/s)		Gear ratio (i)		Efficiency (η) (stage) in (%)	
	normal	extreme		normal	extreme	normal	extreme	from	to
Cylindrical gear	2000	**150000**	**150000**	80	**200**	[1, **800**]	**1000**	97	**99.5**
Planetary gear	5000	35000	(20000)	80	100	[3, 13]	35	98	99.5
Bevel gear	500	4000	50000	40	120	[1, 5]	8	97	99
Hypoid gear	300	1000	20000	30	50	[4, 8]	50	50	< 90
Bevel-cylindrical gear	1000	3000	-	-	-	-	-	-	-
Worm gear	903	1000	30000	25	70	[5, 50]	300	20	97

From the Table 2.1, the cylindrical gears are the most efficient (more than 99 % efficiency a pair gear), have higher power transmission capacity (until 150000kW), higher rotational speed (until 150000tr/min) and higher gear ratio (until 1000). It can be concluded that the cylindrical gear is the better one for the power transmission among all existing types.

2.3.1. Spur gears

The first type of the cylindrical gears is the spur gear, which have teeth parallel to the axis of rotation (Fig. 2.2). According to Dabnichki and Crocombe (1999) the spur gears are the most popular type representing more than 80 % of gear production. Spur gears are used to transmit motion between parallel shafts or between a shaft and a rack. They mainly used when noise generated by the machine is not of the highest importance. In addition, they are commonly used in relatively low power and low rotational speed applications. The spur gear cannot be used in applications where a direction change between shafts is required (Jamali, 2015; Oila, 2003). The common terminology used in spur gears is shown in Fig. 2.2.

Fig. 2.2 Nomenclature of spur gear teeth. Extracted and modified from (Ugural, 2015)



2.3.2. Helical gears

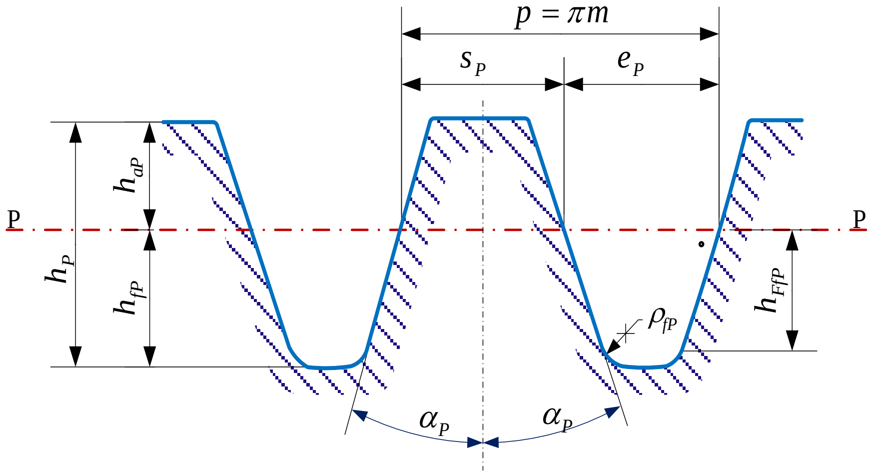
Helical gears resulted from development of spur gears to improve their performance when higher running speeds are required (Jamali, 2015). The teeth are inclined at an angle to the shaft, Fig. 2.3, this angle is usually called the helix angle and it gives the teeth a helix shape. Helical gear teeth are longer than the teeth on a spur gear of equivalent pitch diameter, which makes them, have a greater surface contact than spur gears (contact ratio more then 2). This allows a helical gear to carry more load than a spur gear but, at the same time, it reduces the efficiency relative to a spur gear. They can also be made to mesh with non-parallel axes but the tooth contact is only at a point, which reduces the load capacity. Their major disadvantage is that they are expensive and less efficient than a spur gear of the same size (Oila, 2003). For more details about the basic geometrical relations between various parameters for spur and helical gears we refer the interested readers to the following references (Dufailly, 1997; Henriot, 1979; Nicolet, 2006).

Fig. 2.3 A helical gear

2.3.3. Basic rack

In order to explain the application of the involute method to gears manufacturing, a simple rack is considered as cutting tool, Fig. 2.4. The rack is a degenerated gear, with straight line teeth profile, its primitive is a straight line (P-P), called datum line (Bonori, 2005). The tooth thickness s_P and the space width e_P on the datum line of the basic rack in the reference plane are equal.

Fig. 2.4 Basic rack, extracted and modified from (ISO 53:1998)



There are four types of basic rack tooth profile (A, B, C and D), and their characteristics are given in Table 2.2. In addition, each type of tooth profiles may be used based on application requirements. For example the standard basic rack tooth profile type A is recommended for gears transmitting high torques. For the types B and C are recommended for normal service, and D is recommended for high-precision gears transmitting high torques (see ISO 53:1998 for more details).

Table 2.2 Standard basic rack proportions

Symbol	Types of basic rack tooth profile			
	A	B	C	D
α_P (°)	20	20	20	20
h_{aP}	1m	1m	1m	1m
C_P	0.25m	0.25m	0.25m	0.4m
h_{fP}	1.25m	1.25m	1.25m	1.4m
ρ_{fP}	0.38m	0.3m	0.25m	0.39m

2.3.4. Involute gearing

In case of power transmissions by a gear mating pair, the main basic kinematic requirement to be satisfied by a pair of gear teeth in mesh is the constancy of the instantaneous velocity ratio for gearing (the first low of gearing). In addition, the gear tooth profile should not only provide good strength, long service life and efficiency for the gearing, but also ease of manufacture (Maitra, 1994).

The most commonly used tooth profile which meets these requirements is the involute profile. The involute gear profile originally designed by the mathematician ***Leonhard Euler***, and it's the most commonly used system for gearing today due to its many advantages. The transmission ratio of an involute gear is uniform and insensitive to small fluctuations in the centre distance. Transmission errors are therefore minimized and the gear operation is relatively silent, manufacturing is relatively easy and the same tools can be used to machine gears with different numbers of teeth. The generation tools for involute gears can be produced with high precision. In addition, the profile shift of the involute gear is possible (Heinz and Ralf, 2016; Litvin and Fuentes, 2004).

From the purely geometric point of view, the involute is can be defined as the path of a point on a straight line which rolls without slip along the circumference of a cylinder called the base circle r_b (as shown in Fig. 2.5). In other way, the generation of the involute of a circle can be demonstrated by imagining a cord as it wraps or unwraps around a circle. The involute has many features, for more details we refer the interested reader to (Colbourne, 1987). The main characteristic of an involute that the arc of the base circle (contained), between the point on the base circle and any point of the involute, is equal to the length of a segment of the tangent at the base circle, created by the same point (included) between this point and the tangent point ($\mathrm{arc}MN = \overline{NY}$)

The acute angle α_y formed by the normal and radius-vector at some arbitrary point Y of the involute is called ***the pressure angle***. If the pressure angle is chosen as variable, polar co-ordination of the involute can be defined as following:

$$\overline{YN} = r_b \tan\alpha_y \text{ and } arcMN = r_b \cdot \psi_y = r_b \cdot (\varphi_y + \alpha_y) \tag{2.1}$$

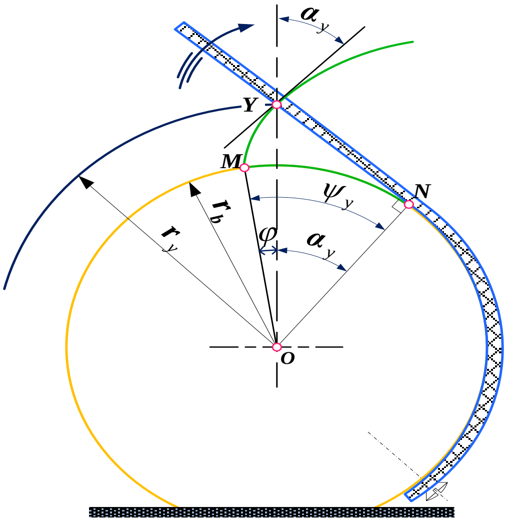
According the above equation:

$$\varphi = \tan\alpha_y - \alpha_y \tag{2.2}$$

The expression on the right side of the equation (2.2) is denoted with $inv\ \alpha_y$ in mathematics, and this is called the involute function of the angle α_y:

$$inv\,\alpha_y = \tan\alpha_y - \alpha_y \tag{2.3}$$

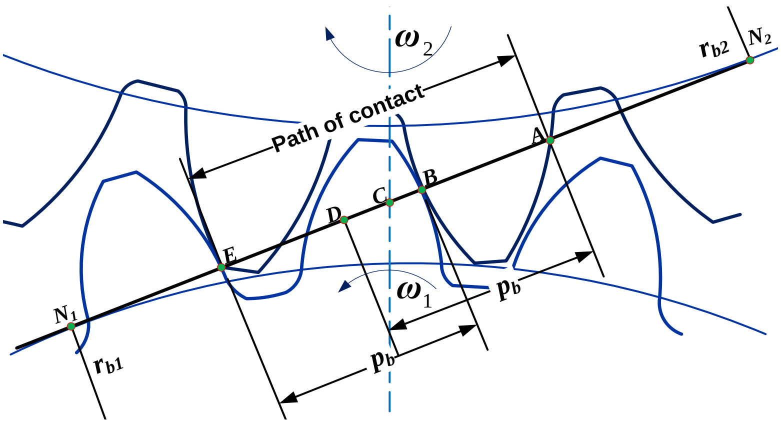
Fig. 2.5 Construction of an involute curve



2.3.5. Characteristic points of contact

Before defining the contact ratio, it is useful to highlight the contact path and their characteristic points. The Figure 2.6 shows the meshing of two spur gears: the two wheels profile mesh each other along the «line of action» also called «line of contact». The tooth contact begins and ends at the intersections of the two addendum circles with the line of action the points A and E.The point B is the point of passing from a period with two teeth pairs in contact to a period of a single meshed teeth pair, C is the pitch point and D is the point of passing from a period of a single meshed teeth pair to a period with two teeth pairs in contact (Atanasovska et al., 2010).

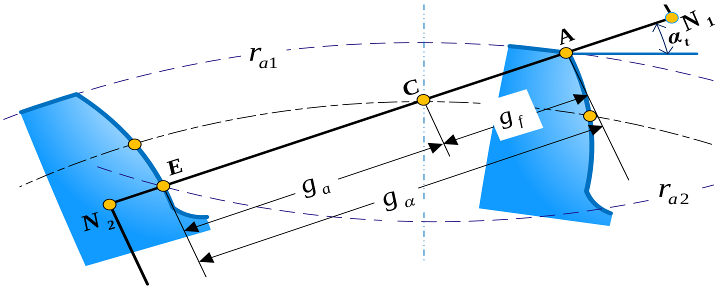
Fig. 2.6 Characteristic points on the path of contact



Practically, before the point A and after the point E the contact between the two teeth cannot take place as the driven gear and driver wheel does not exist respectively. The distance g_α represents the length of path contact, $g_f = \overline{AC}$ represents the approach path length and $g_a = \overline{CE}$ represents the recess path length (Fig. 2.7). So we have:

$$g_\alpha = g_f + g_a \tag{2.4}$$

Fig. 2.7 Contact ratio



2.3.6. Contact ratio

Physically the contact ratio can be defined as a measure of the average number of teeth in contact during the period, in which pair of teeth come into and go out of engagement (Fig. 2.7). For smooth operation and continuous load transfer, contact ratio should be greater than unity. Gears are generally designed with contact ratios of 1.2 to 1.6 (Raul, 2009) for normal gearing and 2 (Henriot, 2007) or more for high contact ratio (HCR). A gear with a contact ratio of 1.0, means that one tooth is just leaving the contact area as another tooth enters the contact area. This is undesirable because it can cause vibrations, noise, and variations in the velocity ratio. Also, the applied load will be applied to the tip of the tooth, creating a large bending moment (Rider, 2015). Following expressions are used for estimating the value of the theatrical contact ratio in case of spur and helical gears.

$$\varepsilon_\alpha = \frac{\textit{length of contact}}{\textit{base pitch}} = \frac{g_\alpha}{p_{bt}} \tag{2.5}$$

For more simplification:

$$\varepsilon_\alpha = \frac{\sqrt{r_{a1}^2 - r_{b1}^2} + \sqrt{r_{a2}^2 - r_{b2}^2} - a_w \sin\alpha_{wt}}{p_{bt}\cos\alpha_{wt}} \tag{2.6}$$

where $r_{a1,2}$ and $r_{b1,2}$ denoting the tip and the base circle radii respectively, α_{wt} is the pressure angle at the pitch cylinder, a_w is the working center distance and p_{bt} is the transverse base pitch. For a helical gear, the contact ratio is the sum of two rapports the transverse contact ratio ε_α and overlap ratio ε_β. So, the total contact ratio ε_γ as following:

$$\varepsilon_\gamma = \varepsilon_\alpha + \varepsilon_\beta \tag{2.7}$$

The overlap ratio ε_β is defined as:

$$\varepsilon_\beta = \frac{b\sin\beta}{\pi m_n} \tag{2.8}$$

where b is the face width, β is the helix angle, and m_n is the normal module.

2.3.7. Interference and undercutting of gear tooth

As reported by Maitra and Prasad (1995), in a pair of gears in mesh, the possibility of the tip of one tooth digging into the portion of the flank of the mating tooth can occur under certain conditions, this effect is called interference. In gear manufacture by the generation process, the teeth of the gear and the cutter are in mesh, like two gears rolling on their pitch cylinders. If interference of cutter occurs with generated gear teeth, a recess is cut in the flank portion of each gear tooth near the root, this phenomenon is termed as undercutting.

Undercutting not only weakens the tooth, but also removes a small involute portion adjacent to the base circle. This loss of involute profile may lead to a serious reduction in the length of contact (Bhandari, 2010). Also, the undercutting of gear teeth reduces the load capacity, causes weakening of the gear tooth and poor functioning resulting in noise, and rapid wear (Mitchiner et al., 1983). The minimum number of teeth to avoid the undercutting of gear teeth is recommended in the literature and in general it is given by an equation.

For spur gears:

$$z_{\min} = \frac{2h_{an}^{*}}{\sin^{2}\alpha_{n}} \tag{2.9}$$

For helical gears:

$$z_{\min} = \frac{2h_{an}^{*}\cos\beta}{\sin^{2}\alpha_{t}} \tag{2.10}$$

where $z_{\min}$ is the limit for the number of teeth to avoid undercut, α_n is the normal pressure angle, α_t is the transverse pressure angle, and h_{an}^{*} is the addendum coefficient.

2.3.8. Sliding phenomenon of gear teeth

The tooth flanks roll and slide on one another. The velocities that occur here are deduced as follows: in the contact point P (Fig. 2.8) of a tooth flank pairing, the resulting velocities of the flank 1 or 2, according to the angular velocities ω_1 or ω_2 and the distances $\overline{N_1P}$ or $\overline{N_2P}$, to the gear centers (Heinz and Ralf, 2016).

$$v_{r1} = \omega_1 \cdot \overline{N_1P} \tag{2.11}$$

$$v_{r2} = \omega_2 \cdot \overline{N_2P} \tag{2.12}$$

The specific sliding is the main geometrical parameter that affects scuffing and the teeth gear wear. It is defined as the ratio between the sliding velocity v_g and the rolling velocity of the mating tooth flanks v_r (Niemann and Winter, 2003). Specific sliding coefficients γ_1 and γ_2 of the mating pinion and wheel are then calculated using the following formulas:

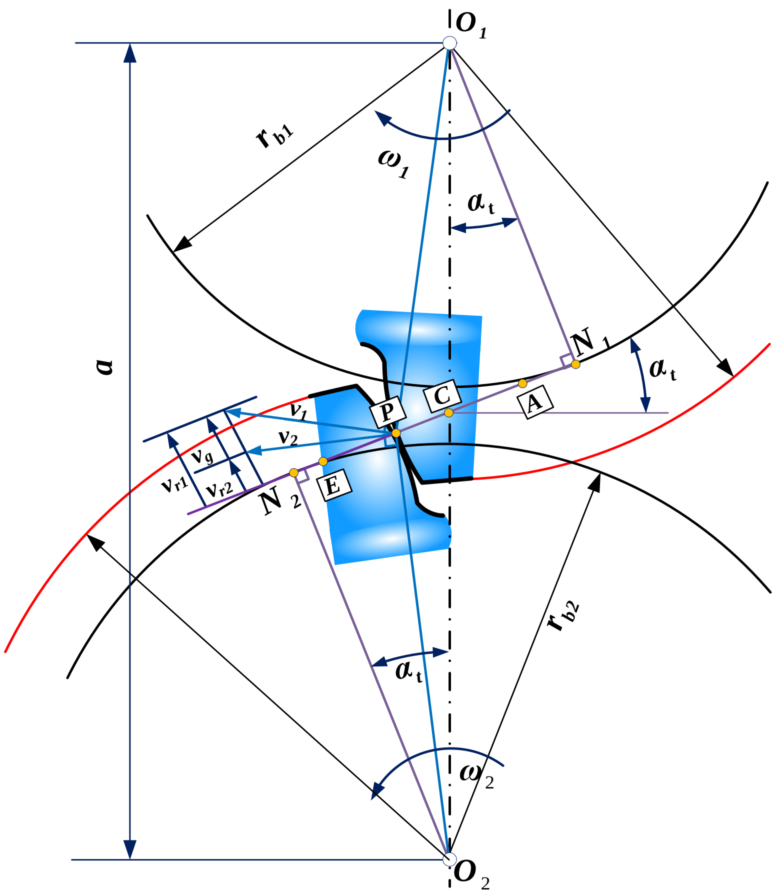
$$\gamma_1 = \frac{v_{r2} - v_{r1}}{v_{r1}} \tag{2.13}$$

$$\gamma_2 = \frac{v_{r1} - v_{r2}}{v_{r2}} \tag{2.14}$$

where v_{r1} and v_{r2} are the rolling velocities between the mating teeth flanks of the pinion and wheel.

In fact, the specific sliding is considered as a function of the position of the point of contact P as shown in Fig. 2.8. At the pitch point C, $v_{r1} = v_{r2}$ so $\gamma_1 = \gamma_2 = 0$. But $\gamma_1$becomes infinitely large in end point N_1 $(v_{r1} = 0)$ and γ_2 becomes infinitely large in end point N_2 $(v_{r2} = 0)$. When v_{r1} and v_{r2} are equal to zero, γ_1 and $\gamma_2$are equal to -1, respectively at $N_1$and N_2.

Fig. 2.8 Sliding as a function of contact point (P)

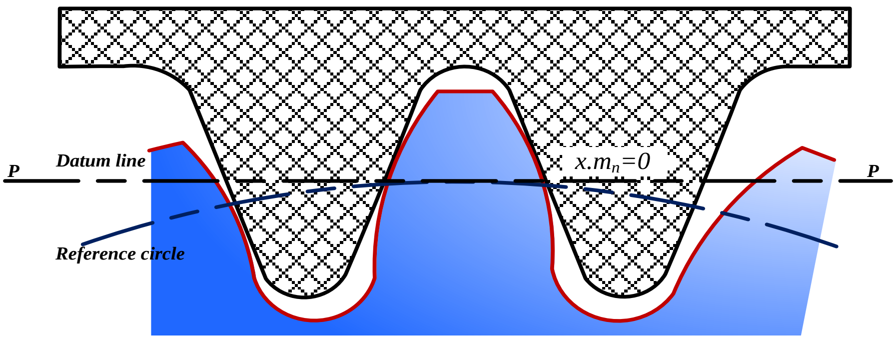


2.4. Profile correction of gears

In gear technology profile correction is variously termed as «addendum modification», «profile displacement», «profile shift», etc. by different authors. Also, the correction factor is called as «addendum modification coefficient» (AGMA 913: 1998; Maitra, 1994). In this work, we shall simply use the terms «correction» and «profile shift coefficient», respectively.

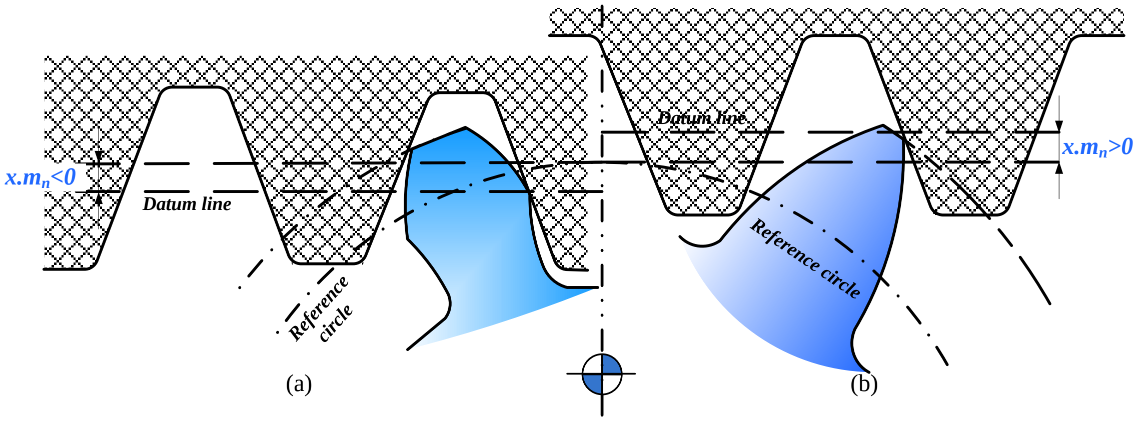
According to the Indian Standard IS 3756: 2002 when gears are produced by a generating process, the datum line of the basic rack profile P-P need not necessarily be tangent to the reference circle of the generated gear as shown in Fig. 2.9.

Fig. 2.9 Gear tooth profile with no addendum modification



The tooth profile and its shape can be modified by shifting the datum line from the tangential position. This displacement makes it possible to use tooth flanks with other parts of the involute curve and different diameters for gears with the same number of teeth, module, helix angle, and cutting tools (Atanasovska et al., 2013). So, the radial displacement from the tangential position is termed as profile shift. Beside, the correction is considered negative when in the direction towards the centre of the gear (Fig. 2.10a) and it is considered as positive when the displacement is away from the centre of the gear (Fig. 2.10b). In other way, the profile shift of the tooth profile is defined as the product of the profile shift coefficient value x and the gear pair module m.

Fig. 2.10 Gear tooth profile: a) with negative correction, b) with positive correction



2.4.1. Geometry of corrected gearing

In standard gearing, also referred to as normal gearing, the gear teeth are generated with standard proportions in the normal manner and without any profile correction or modification. Such gear pairs operate at the standard center distance equal to the sum of the pitch circle radii and standard pressure equal to 20°. As reported by Maitra (1994) in his book «***Handbook of Gear Design***» and based on the correction aspect the corrected gearing can be classified into two broad categories: the «***S_0-gearing***» and the «***S-gearing***».

2.4.1.1. S_0-gearing system

In the first kind of the correction system «*S_0-gearing*» the two components of mating pair of gears receive numerically equal but opposite values of profile shift coefficient. Normally, the pinion is provided with positive correction and the wheel with negative, in other words ($x_1 + x_2 = 0$ or $x_1 = -x_2$). This implies that the pressure angle α_{wt} remains as the standard α_t as in the case of normal gearing. As well as, the working center distance a_w remains unaltered and is equal to the sum of the pitch radii. However, unlike uncorrected gearing system, the datum line of the basic rack profile does not pass through the reference circle of the generated gear in this case. It is shifted away by an amount equal to the numerical value $x \cdot m$ in (mm).

The application of the S_0-gearing (Maitra and Prasad, 1995) is generally recommended where the transmission ratio u is large. Further, the thicker pinion teeth are ensured and the wheel teeth also do not become significantly weak. This system is also sometimes recommended where for certain specific reasons the normal tooth-thickness of the gear pair or the specific sliding velocities between the meshing teeth flanks are to be changed. The S_0-gearing system tends to equalize the tooth strength and thereby reduces the susceptibility to such damage.

Furthermore, this type of gearing is used if the number of the teeth of pinion is less than the particle limiting number of teeth to avoid undercutting $z_{\min}$. Also, the sum of both gears should be equal to or greater than twice the limiting minimum number of teeth $z_1 + z_2 \geq 2 \cdot z_{\min}$ (Henriot, 2007).

2.4.1.2. S-gearing system

In the second type of the correction «*S-gearing*», the sum of the profile corrections of the two mating gears is not equal to zero; it is either positive or negative. Usually the sum is positive in almost cases in order to take advantage of the beneficial effects of positive correction. In general, the sum is so divided that the pinion gets the bigger share of the positive correction. In S-corrected gearing system gears operate on different center distance and pressure angle.

The circles of the two gears which touch at the pitch point are termed as «working circles». The tooth pressure line which is always tangent to both the base circles and which passes through the pitch point, now makes an angle α_w instand the pressure angle α_t. This angle is termed as the «working pressure angle», in other word $\alpha_t{}' \neq \alpha_t$ and $a' \neq a$.

For spur gears:

$$inv\,\alpha' = \frac{2\tan\alpha_n\left(x_1+x_2\right)}{\left(z_1+z_2\right)} - inv\,\alpha_n \tag{2.15}$$

For helical gears:

$$inv\,\alpha_t' = \frac{2\tan\alpha_n\left(x_1+x_2\right)}{\left(z_1+z_2\right)} - inv\,\alpha_t \tag{2.16}$$

where z_1, z_2 are the teeth number of pinion and wheel respectively, x_1 is the profile shift coefficient of pinion and x_2 is the profile shift coefficient of wheel.

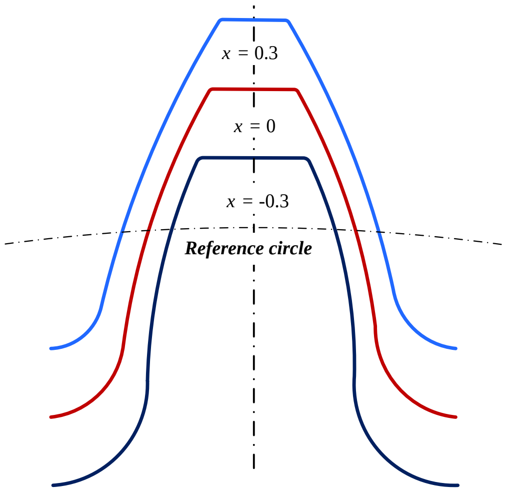
For more details about the geometry calculation of gear pair shifted by the S_0-gearing or the S-gearing systems, the reader can refer to the standards (DIN 3960:1987; ISO 21771:2007) and (AGMA 913: 1998), respectively. In the present thesis the standard DIN 3960:1987 is used for all tooth geometry calculation.

2.4.2. Importance of profile shift of the teeth

The first use of profile shift was to avoid undercut in gears with less than seventeen teeth. However, it was later realized that profile shift improves most aspects of gear operation, and today it is used for many reasons including the kinematic conditions and the working performances of the gear meshing (Yu et al., 2016). The gear correction is carried out in order to achieve of non standard distances between shafts, to improve the loading capacity and to avoid narrow top lands of teeth. The profile shift is generally recommended for balancing specific sliding to maximize wear resistance and Hertzian fatigue resistance; balancing flash temperature to maximize scuffing resistance; balancing bending fatigue life to maximize bending fatigue resistance.

Further, the value of profile shift of teeth depends solely (IS 3756: 2002), during manufacture, on the relative position of the gear to be cut and the cutter. Hence, the influence of profile shift on the cost of toothed gears is nil in the majority of cases. The Figure 2.11 illustrates the tooth profiles of the driving gear with three profile shift values (x = 0.3, 0 and -0.3).

Fig. 2.11 Effect of profile correction on the shape of teeth



From the Figure 2.11, it can be seen that gear teeth with a positive correction causes an increase in root thickness and a decrease in tip thickness. In this case the tooth is more resistant to the root stress, but also becomes more pointed. Similarly, the negative shifted of the teeth has the reverse effect on tip and root thickness. In addition, an increase in the profile shift coefficient of the teeth of the mating gears causes a slight decrease in the contact ratio of the gear pair.

2.4.3. Effects of the profile shift on the main parameters of gears

Gear tooth profile has an immense effect on the main operating parameters of gear pairs. By the suitable modification of the tooth profile, the vibration and noise of gear system can be reduced greatly. In literature, a substantial amount of published researches can be found, addressing the effects of the profile shift on the main parameters of spur and helical gears, such as the bending strength, surface durability and dynamic vibration.

For example, Oda and Shimatomi (1980a) studied the effects of the profile shift on bending fatigue strength in cylindrical involute gears. They analyzed different spur gears, made of normalized steel, casehardened and spur gears with higher pressure angle (Oda and Shimatomi, 1980b; Oda and Tsubokura, 1981). The investigation was performed based on the theoretical and experimental aspects. Later, Oda and Koide (1984) presented a study on the profile shift's effects on root stresses of helical gears. Bending fatigue tests were performed with several amounts of profile correction. In addition, the validity of ISO and JSME formulas for bending strength of profile shifted gears was examined by using a hydraulic bending fatigue testing machine. It was found that due to the increase of the positive profile shift coefficient, the bending fatigue limit loads of helical gears is much smaller than that in the case of spur gears.

Kuang and Yang (1989) in their paper introduced a semi-empirical equation to evaluate the fillet stress concentration for spur gears cut by a profile shifted cutter. It was found that gear tooth surfaces cut by a positive shifted cutter can improve the undercutting phenomena. In the same way, Durmus et al. (1996) investigated the effects of the profile shift coefficient on the root stresses of spur gear by using the FEA. The study considers the positive and negative correction gears. It was found that the stress concentration factor increases with an increasing profile shift coefficient due to a decrease in the radius of curvature at tooth fillet.

Furthermore, the FE method has been used also by Atanasovska and Vera (2006) to study the influence of profile shift on the load capacity of spur cylindrical involute gear. In order to compare the stress states of meshed teeth's flanks and roots during the meshing period for gear pairs with different values of profile shift coefficients, the comparative diagrams are made and shown. It was found that the correct selection of profile shift coefficients can directly affect on improvement of characteristics of gear pair working that guide to increasing gear life-time, also it reduces vibrations and shocks during working of gear pair.

Miriča and Dobre (2007) investigated the aspects of the distribution of the profile shift coefficients between mating gears according to the standards recommendations of ISO/TR 44674: 1982, DIN 3992: 1964 and PD 6457: 1970. Further, Shuting (2008) investigated the effect of correction on tooth contact strength, bending strength and other performance parameters of spur gears, by using mathematical programming and finite element method. Effects of profile shift and contact ratio on gear strength and basic performance parameters were also discussed. In addition, the strength calculations of HCR gear by considering misalignment error and lead crowning were presented in this work.

Atanasiu and Doroftei (2008) presented a dynamic tooth load analysis of spur gears with profile correction. In this study, the dynamic model includes the non-linear time varying mesh stiffness, variable tooth profile errors, and tooth profile coefficients. The results showed important effects of the coefficients on the dynamic load variation.

In Imrek and Unuvar (2009) a practical investigation of the load and velocity effects on scoring resistance of corrected spur gears is presented. The experimental study is based on works were performed in a FZG* (Gear Research Centre) gear test apparatus, and the scoring was investigated by using different speed and load levels with gear pairs having different profile shift. It was found that wear at profiles with negative correction was higher than the profiles having positive one. Baglioni et al. (2012) in their study analyzed the efficiency of shifted spur gears for various operating conditions and gear geometry. Four different allocating methods of profile shift coefficients are compared

*FZG: Forschungsstelle für Zahnräder und Getriebebau including the standards recommendations of DIN 3992: 1964, Maag (1990) and two other methods given in (Niemann and Winter, 2003). The authors suggested that increase in sum of addendum modification results to increase the gear efficiency. Furthermore, Diez-Ibarbia et al. (2015) studied in their paper the influence of correction on the energy efficiency of spur gears. The developed load contact model considers a shifting profile under different operating conditions. The authors concluded that an increase in the profile shift influences the load sharing properties of gear pair, thus lowering the transmission efficiency. Besides, Zaigang et al. (2016) studied the influence of profile shift on the internal spur gear pair mesh stiffness. The results showed that with an increase in the value of profile shift coefficient, the single-tooth mesh stiffness also increases, while the multi tooth stiffness decreases, influencing dynamic responses.

In addition, Atanasovska (2016) analyzed the influence of profile shift on spur gears stability. The author noted that the application of such approach will lead to the increase in the energy efficiency and stability of the whole power transmission system. Yu et al. (2016) investigated the analytical relationship between the drive-side and back-side mesh stiffness for spur gear pairs with various profile shift. Further, a time-varying asymmetric mesh stiffness model was built and employed in two typical gear dynamic models to simulate the effect of the correction on gear dynamics.

2.5. Survey of profile shift coefficients

Nowadays the increasing demand for high strength, efficient, quiet and high precision gear design leads to various methods for improvements. One of the most widely used methods is tool shift, which enables to achieve of non standard distances between shafts, to improve the loading capacity and to avoid undercutting in the gears with small numbers of teeth. Additionally, the profile shift has an immense effect on the main operating parameters of gear pairs (working life, efficiency, vibrations, power losses, noise and wear); it can be used in order to optimize the design, according to the special requirements of each problem. Consequently, it becomes clear that the rational choice of the profile shift coefficients is one of the most important stages in gear drive design (Maitra, 1994; Jelaska, 2012).

In recent decades, a variety of methods have been made for determining the profile correction values in order to satisfy various optimization criteria. Indeed, different standards and recommendations from manufacturers are developed in order to provide a good choice distribution of profile shift coefficients between mating gears. For instance, the DIN 3992: 1964 standard provides a diagram for the total shift (as shown in Fig. 2.12.a) and some other ones for sharing it between the pinion x_1 and the wheel x_2 (as shown in Fig. 2.12.b and Fig. 2.12.c), which give teeth with balanced sliding velocity and stress at the root. Furthermore, the British Standard Institute guide PD 6457: 1970 suggests the use of computation formulae for the profile shift. Three cases were considered in this method.

- General applications:

$$x_1 = \frac{1}{3}\left(1-\frac{1}{u}\right)+\frac{\Sigma x}{1+u} \tag{2.17}$$

- Equalizing bending stresses:

$$x_1 = \frac{1}{2}\left(1-\frac{1}{u}\right)+\frac{\Sigma x}{1+u} \tag{2.18}$$

- Balancing specific sliding:

$$x_1 = \frac{1}{\sqrt{z_{v1}}}\left(1-\frac{1}{u}\right)+\frac{\Sigma x}{1+u} \tag{2.19}$$

where u is the transmission ratio and z_v is the virtual number of teeth.

For the same manner ISO/TR 44674: 1982 standard provides a practical guide, as shown in Fig. 2.13, for the value limits and distribution of the correction in case of external parallel axis cylindrical involute gears for speed-reducing and speed-increasing. The recommendation takes into account great contact ratio, minimum shocks and general applications. The ISO method defines the pinion profile shift in the form:

$$x_1 = \lambda\frac{z_2 - z_1}{z_2 + z_1} + \Sigma x\,\frac{z_1}{z_2 + z_1} \tag{2.20}$$

where λ is a factor adopted in the range [0.5, 0.75] for speed reducing gears and in the range [0, 0.5] for speed increasing gears. In the case where the gear ratio exceeds 5, the allocation of the sum of the shift is calculated with $u = 5$.

The American Gear Manufacturers Association AGMA 913: 1998 recommends the choice of the criterion considered of greatest importance for practice. It is the reason why the problem of the distribution of the shift coefficient sum on each gear wheel could be met in the case of the largest used thermal treatment for cylindrical gears of general use: case hardening. Moreover, DIN 3994 and DIN 3995 standards recommend uniform values ($x_1 = x_2 = 0.5$) aiming to increase the load capacity. The disadvantage of this recommendation is the generalization of center distance corrections which often leads to a significant reduction in the contact ratio.

The allocation of the total shift can also be performed according to the experience or the recommendations of some prominent machine building company (Jelaska, 2012). For example, the German company FZG provides formula (Eq. 2.21) for gears with great loading capacity, minimum thickness at the tip and contact ratio greater than one. Maag (1990) recommendation cited by Baglioni et al. (2012) suggests that apportionment of the profile shift coefficients is adequate for a satisfactory compromise between the tooth root thickness and maximum gear efficiency. The manner to distribute the shift coefficient sum between the mating gears is well explained in Maag book (Maag, 1990), where a graphical system can be used to approximately apportion this sum or by using a formula (Eq. 2.22).

$$x_1 = \frac{\Sigma x}{u+1} + \frac{u-1}{u+1+0.42 z_{n2}} \tag{2.21}$$

$$x_1 = \frac{\Sigma x}{2} + \left(A - \frac{\Sigma x}{2}\right)\left(\frac{\log(u)}{\log\left(\frac{z_1 \cdot z_2}{100}\right)}\right) \tag{2.22}$$

where for a pressure angle $\alpha_n = 20°$ the relative coefficient is $A = 0.50$.

Some graphical charts can be found in the literature for determining the profile shift coefficient values. Indeed, Henriot (1979) proposed a graphical system (Fig.2.14) required for balanced specific sliding in both gears, which reduces wear and heavy scoring risks at the extremes of contact path. The graphical system was divided into two zones, the first is S_0-gearing system for $z_1 + z_2 \geq 60$ and the second zone is S-gearing system for $z_1 + z_2 < 60$. The profile shift values are defined in terms of the pinion teeth number and the transmission ratio. However, this system was reserved only for gears with 20° pressure angle.

In addition, Jelaska (2012) presented two graphical approaches for choosing the profile shift coefficients values. The first one based on using so-called diagrams of blocking contours in (x_1, x_2) coordinates. As mentioned by the author, various isograms that characterize a gear pair can be added to block-contour: for example isograms of equal specific sliding on the meshed teeth flanks, isograms of constant root or contact stresses, etc. The second one used so-called lines of pairs in the (z, x) diagram based on the FZG and ISO standards recommendations. Both methods were well detailed and explained by several application examples.

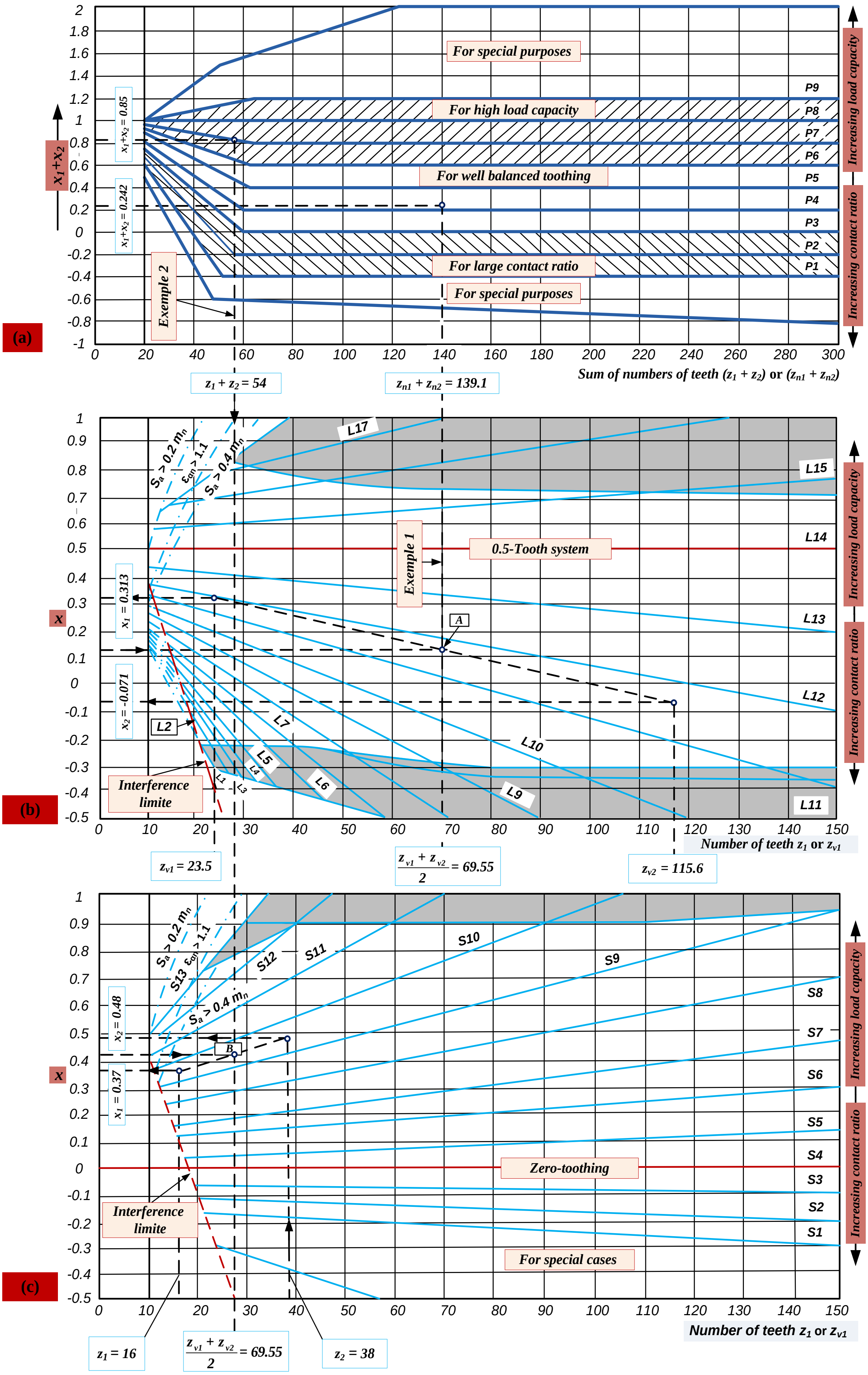


Fig. 2.12 Graphical method of DIN 3992 for distribution of the sum of profile shifts on mating gears. Extracted and modified from (DIN 3992:1964)

Fig. 2.13 Conventional and recommended limits for the sum of the profile shift coefficients and zones for special cases. Extracted and modified from (ISO/TR 44674: 1982)

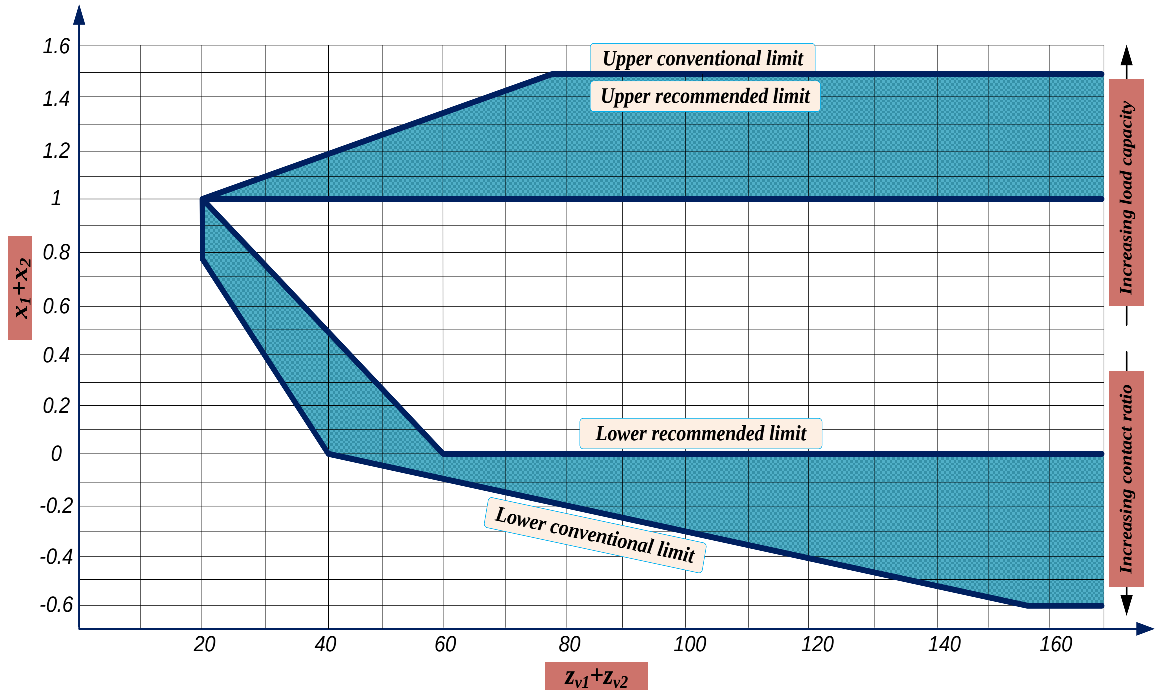
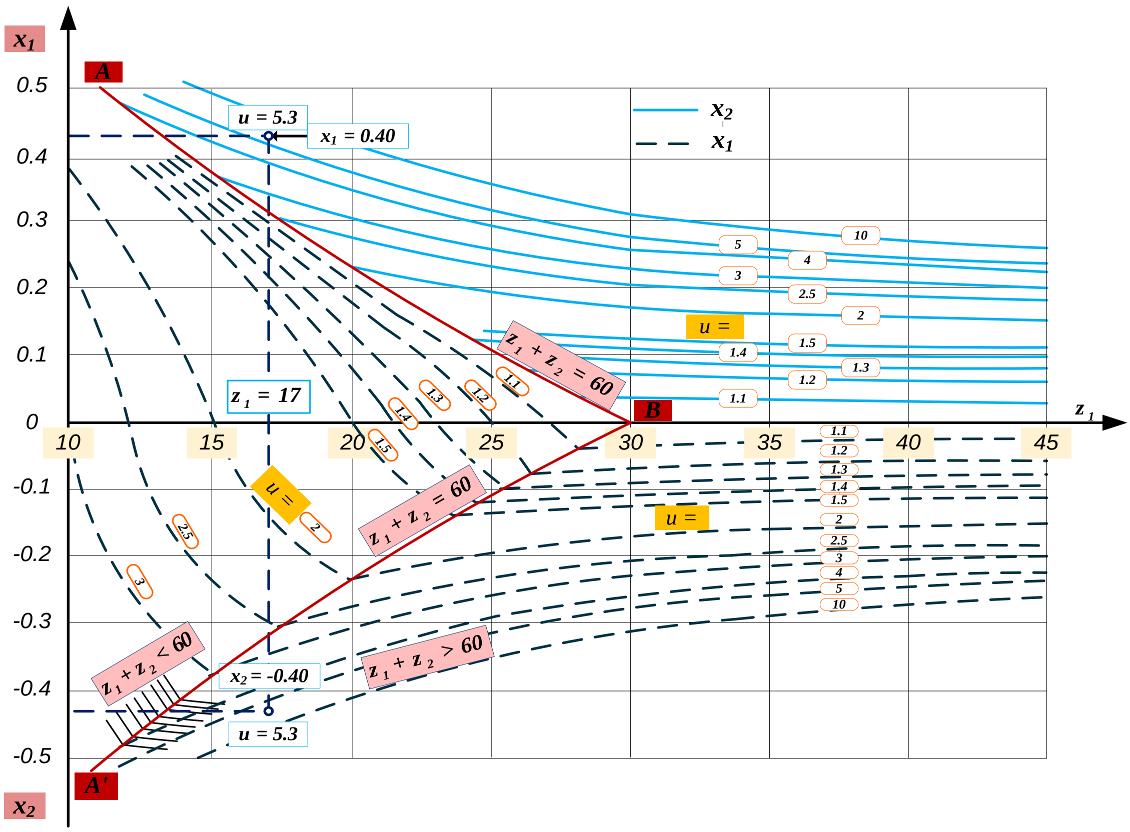


Fig. 2.14 Graphic representations of the profile shift coefficients required for balanced specific sliding in both gears. Extracted and modified from (Henriot, 1979)



Therefore, the shift coefficients values read from a graph or using approximate formulas are not accurate and interpolations increase the error. Consequently, the obtained results are not very reliable. To overcome the limitations in the above methods Pedrero and Artés (1996) and Pedrero et al. (1996) developed simple analytical models to estimate the approximate equations of the profile shift coefficients. The equations gives the relation between the shift coefficients of gear pairs and other conditions, such as equalizing specific sliding or ensuring pre-established values for the center distance. The proposed models are valid for every value of the pressure angle and the addendum. The developed method requires neither iterations nor tabular values, and makes it efficient for computer applications.

Some studies suggest the use of an optimal design methodology. Indeed, a tooth-geometry optimization feature using an adaptive-grid-refinement (AGR) algorithm procedure has been proposed by Samo et al. (2001). The optimization purposes were to satisfy compromise between specific sliding and Hertz pressure. In Spitas and Spitas (2007) non standard involute gears with optimized tooth geometry for minimum root stress are presented. The optimization procedure was based on the complex algorithm. The root stress was the objective function and the active constraints include all of the kinematical, manufacturing and geometrical conditions. The geometrical design parameters for the gear pair were the thickness coefficients for both gears and the profile shift coefficients. The results found in this paper indicate that optimal designs can achieve up to 8.5 % reduction of the fillet stress.

The same authors Spitas and Spitas (2012) discussed in their study the effect of combined profile correction and changes in nominal tooth thickness of meshing gears on the minimization of the root bending stress. A total of seven nonlinear inequality constraints were considered, including: allowable profile shift. A design space is defined in terms of the profile shift and the thickness coefficients. The optimization problem was solved by employing complex algorithm method. In order to treat the constraints functions the authors used the penalty approach. The optimum design was verified using two dimensional photo elasticity on PCB plastic gear tooth models.

Antal (2014) presented in his paper an iterative method implemented and programmed in MATALB, in order to optimize the profile shift values for the cylindrical spur gears. The objective was the equalization of the powers lost by friction, between the teeth's flanks, during the meshing of the teeth. The same author Antal (2015), using the same approach in (Antal, 2014), optimized the profile shift coefficients this time to equalize the spur gear efficiency at the extreme points of contact path. The solutions were obtained for different pairs of teeth numbers.

Some of the more advanced optimization techniques, such as GA and Generalized Particle Swarm (GPSO) algorithm have been applied also in order to determine the optimum correction to satisfy various optimization criteria. In fact, Zhang and Wen (2002) used the GA with optimization purpose of the optimum profile shifts coefficients of cylindrical gear pair having hard tooth flanks in a closed drive. The objective function was to equalize the form factors of the gear pair. Moreover, the proposed optimization procedure takes into account the different geometrics condition of the gear design such as gear-tooth thickness, contact ratio, and tooth interferences. Furthermore, in Antal (2009) a new algorithm for designing helical gears with profile shift coefficients is introduced, based on GA toolbox of MATLAB. The optimization procedure was developed based on the principle of equalized the maximum specific sliding coefficients at the extreme points of the meshing.

Recently, Van-Thoan et al. (2015) investigated the optimal choosing of the profile shifts coefficients for external involute spur gear by using the GA, based on all the necessary conditions according to the standard design. Three objectives were considered the maximum of sum of the profile shift coefficients, balancing specific sliding coefficients and balancing the form factors of the gear pair. Very recently, Miler et al. (2017) used the GA to solve the optimization problem of spur gear. The authors considered the pinion and wheel profile shifts as variables additionally to the module, the face width, and the number of teeth. The gear pair volume served as a fitness function, while the tooth root bending strength and the contact pressure calculations were used as constraints. Optimization results of the pair without profile shift and the arbitrary profile shifted pair were compared. The solutions found by genetic algorithm were verified using commercial gear optimization software.

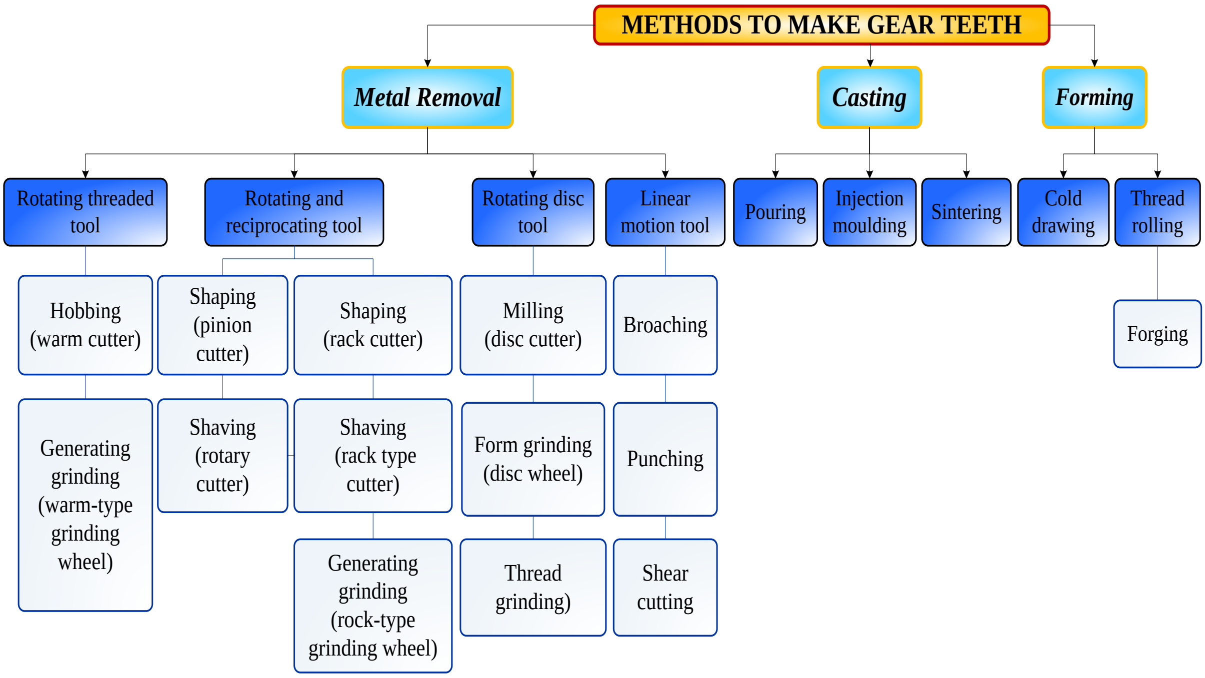
GPSO was used for solving the problem of the optimal high contact ratio (HCR) gearing geometry in order to achieve a balance among contact ratio, sliding velocity, bending strength, pitting damage, and scuffing phenomenon (Rackov et al., 2014; Rackov et al., 2017; Rackov et al., 2018; Veres and Bosansky, 2017). The design variables were already the profile shift coefficients and the addendum coefficients of the gear pair.

2.6. Gear manufacturing

Depending on the application of the gears, the manufacturing method used for the forming of gear teeth can be identified (Raul, 2009). There are usually various methods employed to manufacture the gear teeth. They can be classified into three categories: casting, forming and metal removal (Fig. 2.15). Firstly, gears can be formed by using different casting processes such as sand casting and shell molding, and these processes in general need minor finishing operation, depending on the quality of the die, to obtain the required accuracy of gears (Swift and Booker, 2003). The second kind of gear manufacturing is the forming method, which can be divided into two types: cold drawing and thread rolling processes. In forming processes, the blank gear is passed against series of forming rolls to form the shape of teeth gradually until the required tooth profile is obtained. The rolling process method is used to improve metal mechanical properties whereby tooth profiles of a high quality can be obtained. The gears manufactured by forming processes are more accurate than those by casting processes (Stephen, 2016). The third method used for gear machining is the metal removal including hobbing, shaping (using rack cuter or disc cuter), milling, and broaching (Mohammed, 2014).

The highly stressed gears are usually made of steel and then cut with form or generating cutters (Raul, 2009). The shape of the form cutter is the exact tooth space between gear teeth. In generating cutters, the tool has the different shape compared to the tooth profile, and this is moved relative to the gear blank, so that the required gear tooth shape is obtained after a full revolution of the gear blank. Generating gears from gear blanks is one of the most accurate ways of obtaining high-precision gears; this is mainly due to the stiffness between the gear blank and the cutter during the generating procedure.

Fig. 2.15 Gear manufacturing methods (Stephen, 2016)



2.7. Conclusion

This chapter provided a fundamentals review of the cylindrical spur and helical gears with the involute profile. The chapter begins with the basic classification of the gears, and the nomenclature geometry of the cylindrical. In addition, the geometry of corrected is discussed.

CHAPTER THREE:

Mathematical optimization:

State of the art

MATHEMATICAL OPTIMIZATION: STATE OF THE ART

3.1. Introduction

One of the most fundamental principles of our world, which also occupies an important place in the computer world, industrial, etc., is the search for an optimal state. Indeed, many scientific, social, economic and technical problems have parameters that can be adjusted to produce a more desirable result. These can be considered as optimization problems and their resolution is a central topic in operational research.

Therefore, this chapter firstly introduces the main definitions related to optimization problems. Then principles classification of optimization methods is presented. In general, there are two classes of optimization methods for solving continuous and mixed discrete design problems: stochastic and deterministic. The most techniques used to handle with constraints are presented and explained. In addition, some concepts and definitions related to multi-objective optimization problems are presented. Finally, the remainder of the chapter is reserved for a detailed overview of the differential evolution algorithm.

3.2. Optimization problem

An optimization problem can be static or dynamic, mono-objective or multi-objective (i.e. several objective functions must be optimized). In most optimization problems, constraints may be imposed by the characteristics of the problem. These constraints have to be satisfied in order to achieve an acceptable solution. Thus, these problems are called constrained optimization problems (COPs). COPs can be classified in several ways depending on their mathematical properties (Mohamed, 2017). They can be classified based on the nature of variables, such as real, integer and discrete, and may have equality and/or inequality constraints. Another important alternative classification depends on the type of expression of objective and constraints functions. It can be classified as linear or non-linear, uni-modal or multimodal. So, any constrained optimization problem, in general, can be mathematically expressed as follows:

$$\text{Find } x \text{ which minimizes or maximizes } f(x) \tag{3.1}$$

$$\text{Subject to: } \begin{cases} g_i(x) \le 0, \quad i = 1,\dots,m \\ \text{and} \\ h_j(x) = 0, \quad j = 1,\dots,q \end{cases} \tag{3.2}$$

where $x \in \mathbb{R}^n$ is the vector of solutions $x = [x_1, x_2, ..., x_n]^T$, where each $x_i, i = 1, \dots, n$ is bounded by lower and upper limits $L_i \leq x_i \leq U_i$ which define the search space, $g_i(x) \leq 0, i = 1, ..., m$ is the number of inequality constraints and $h_j(x) = 0, j = 1, ..., q$ is the number of equality constraints.

3.3. Optimization methods classification

A large variety of optimization algorithms exist which have been developed in order to solve the COPs. There are many ways for classifying these algorithms, based on the problems that they were designed for, and based on the algorithms characteristics themselves (Dréo, 2003). Firstly, the optimization problem can be static or dynamic. Depending on the nature of the variables, the problem can be discrete or continuous. On the other hand, the optimization methods can be deterministic or stochastic.

3.3.1. Deterministic optimization algorithms

Deterministic optimization algorithms are the classical branch of optimization in mathematics. They get the same solution with the same number of objective function evaluations, of course if the search space, starting-point, and termination conditions are unvaried. In other words, if the algorithm is run multiple times on the same computer, the search time for each run will be exactly the same (Cavazzuti, 2013; Qing, 2009). Mainly the deterministic optimization methods can be divided into two broad classes (Saravanan, 2006): direct search and gradient search methods. One common characteristic of most of these methods is that they all work on a point-by-point basis. In addition, an algorithm starts with an initial point (usually supplied by the user).

3.3.1.1. Direct search algorithms

In computer science, the direct search is also termed as «zero-order» and «derivative free» algorithms by different authors. This type of algorithms does not require any information about the gradient of the objective function for solving optimization problems. The main principle of a direct search algorithm is searches a set of points around the current point, looking for one where the value of the objective function is lower than the value at the current point (Hooke and Jeeves, 1961). The well-known examples of some of the original direct search methods include the pattern search method (Hooke and Jeeves, 1961), the conjugate direction method and the simplex method of Nelder and Mead (Nelder and Mead, 1965). In general, direct search methods are expensive and seem to work on simple unimodal functions. These methods are indeed incapable of exploring the search space and can stay confined in the first local minimum they reach (Saravanan, 2006).

3.3.1.2. Gradient based algorithms

The second class of the deterministic optimization algorithms is gradient based methods, which require the knowledge or gradients of functions and constraints, either exactly or numerically. The well-known examples of some gradient based algorithms include the Newton-Raphson algorithm, the quasi-Newton algorithm, the gradient-based algorithm (Ravindran et al., 2006).

Most deterministic algorithms are not guaranteed to find the global optimal solutions because these algorithms usually terminate when the gradient of the objective function is very close to zero, which may happen both in case of local and global solutions (Saravanan, 2006). The other difficulty is the calculation of the gradients themselves. In most real-world engineering design problems, the objective function is not explicitly known. Some simulation (running a finite element package, for example) is required to evaluate the objective function or constraints (Qing, 2009).

3.3.2. Stochastic optimization algorithms

These methods have a great ability to find the overall optimum of the problem. Unlike most deterministic methods, they do not require either a starting point or knowing the gradient of the objective function to achieve the optimal solution. Stochastic optimization algorithms can be divided into two major categories according to their origins: physical algorithms and evolutionary algorithms (Qing, 2009).

3.3.2.1. Physical-based algorithms

In these algorithms, search agents communicate and move throughout the search space according to physics rules such as gravitational force, electromagnetic force, inertia force, and so on (Dhiman and Kumar, 2017). The Monte Carlo (Gentle, 2003) and the SA algorithms (Kirkpatrick et al., 1983) are two of the most prominent algorithms in this category. As well, other physics-based techniques are devlopped in few recent years, such as Gravitational Search Algorithm (GSA) (Rashedi et al., 2009), Colliding Bodies Optimization (CBO) (Kaveh and Mahdavi, 2014), Black Hole (BH) (Hatamlou, 2013), etc.

3.3.2.2. Evolutionary algorithms

Evolutionary Algorithms (EAs) are one of the most studied population based methods. They are inspired from the process of natural evolutionary principles (Darwin, 1859) in order to develop search and optimization techniques for solving complex problems. The Darwinian Theory explains the principle of natural selection, which favors the survival of species that are matched with their environmental conditions and explains the evolution of species as an outcome of stochastic variations and natural selection (Back, 1996). Due to their abilities to tackle complex and real-world optimization problems in many different application areas, EAs have gained significant amount of research interest over the last few decades.

The EAs are population-based meta-heuristic optimization algorithms that use biology-inspired mechanisms like mutation, crossover, natural selection, and survival of the fittest in order to refine a set of solution candidates iteratively (Boussaïd et al., 2013). In short, the procedure optimization of a standard EA is presented in Algorithm 3.1.

Algorithm 3.1: Generic evolutionary algorithm

$G := 0$
Generate an initial population: $P^{G=0}$ with random individuals
Evaluate fitness of all initial individuals
while *stopping criterion is not satisfied* **do**
  Perform the recombination
  Perform the mutation
  Evaluate the new individual
  Perform the selection
  $G := G + 1$
end

As reported by Boussaïd (2013), an individual represents a potential solution to the problem being solved. Firstly, the population is generated randomly within the boundary constraints of each variable. Then each individual in the population is evaluated by a fitness function, which is a measure of quality with respect to the problem under consideration. At each iteration or generation, a population of candidate solutions is capable of reproducing and is subject to genetic variations followed by the environmental pressure that causes natural selection (survival of the fittest). New offspring solutions are produced by recombination of parents and mutation of the resulting individuals to promote diversity. A suitable selection strategy is then applied to identify the solutions that survive to the next generation. This process repeats until a predefined number of generations or some other specific stopping criteria are met.

During the last decades, a huge number of meta-heuristic algorithms have been proposed in order to solve various problems in different fields, among them GA (Holland, 1975), PSO (Eberhart and Kennedy, 1995), DE (Storn and Price, 1995), Harmony Search (HS) (Lee and Geem, 2004), Ant Colony Optimization (ACO) (Dorigo and Birattari, 2006), Artificial Bee Colony (ABC) (Karaboga and Basturk, 2007), Firefly Algorithm (FA) (Yang, 2009), and more recently Water Cycle Algorithm

(WCA) (Eskandar et al., 2012), Whale Optimization Algorithm (WOA) (Mirjalili and Andrew, 2016), etc.

3.4. Constraint handling in evolutionary algorithms

In the real world, the problems are often subject to different types of constraints which have to be satisfied in order to achieve an acceptable solution. For this reason, several methods have been developed to deal with the constraints functions, and these techniques can be broadly classified into four categories: 1) penalty functions, 2) decoders, 3) special operators, 4) separation of objective function and constraints. For more detail, there is a survey reviews focused on constraint handling techniques in the EAs (Coello, 2002; Mezura-Montes and Coello, 2011). Below, we will briefly describe the most widely used handling techniques.

3.4.1. Penalty functions

Probably the most common approach used in the EAs to deal with constraints is the penalties, due to its simple principle and easy implementation. Penalty functions were originally proposed by Courant in 1940 (Courant, 1943). The idea of this method is to transform a constrained optimization problem into an unconstrained one by adding (or subtracting) a certain value to/from the objective function based on the amount of constraint violation present in certain solution. In classical optimization, two types of penalty are considered: interior and exterior. The exterior penalty is the most used because don't require an initial feasible solution (Coello, 2002). Mathematically, it can be formulated as follows:

$$F(x) = f(x) + p(x) \tag{3.3}$$

where $F(x)$ denots the expanded fiteness to be optimized and $p(x)$ is the penalty function that can be calculated as follows:

$$p(x) = \sum_{i=1}^{m} \lambda_i \cdot \max\left(0, g_i(x)\right)^2 + \sum_{j=1}^{q} c_j \cdot \left|h_j(x)\right| \tag{3.4}$$

where λ_i and c_j are positive constants named penalty factors.

Really their implementation is quite simple, however, deciding optimal values of penalty terms especially for the optimization problems with highly constraints turns out to be a difficult optimization problem itself (Runarsson and Yao, 2000). Several techniques with different approaches have been proposed in the literature to avoid this dependency of the values of the penalty factors and the most common are the following: death penalty, static penalties, dynamic penalties, annealing penalties, adaptive penalties, and so on (Montes, 2004).

3.4.2. Separation of objective function and constraints

Unlike penalty functions which combine the value of the objective function and the constraints of a problem to assign fitness, these approaches handle constraints and objectives separately. The most representative approaches, which work based on this idea are: the methods based on the superiority of feasible points (Deb, 2000) and the methods based on evolutionary multi-objective optimization concepts (Coello and Montes, 2002).

The first method of this category is the three feasibility rules, which was originally proposed for binary selection tournaments in a GA (Deb, 2000). The approach based on three selection criteria to handle the constraints, as in the following:

1. Between two infeasible solutions, the one with smaller degree of constraint violation is preferred;
2. If one solution is infeasible and the other one is feasible, the feasible solution is preferred;
3. Between two feasible solutions, the one with better objective function value is preferred.

The sum of constraint violation can be calculated as follows:

$$\phi(x) = \sum_{i=1}^{m} \max\left(0, g_i(x)\right)^2 + \sum_{j=1}^{q} \left|h_j(x)\right| \tag{3.5}$$

Several authors have adopted multi-objective optimization concepts to solve constrained optimization problems (Wang et al. 2016). In this case, the objective functions and the constraints are also handled separately. These approaches are reviewed in (Coello and Montes, 2002) and can be divided in two groups: techniques which transform a COP into a bi-objective problem. The second techniques transform a COP into a multi-objective optimization problem (the original objective function and each constraint are handled, each, as a single objective function).

3.5. Multi-objective optimization

In many real-world applications, it is not a question of optimizing only one criterion but rather of simultaneously optimizing several criteria and which are generally conflicting. In gear design problems, for example, it is most often necessary to find a compromise between kinematic, dynamic and geometric performances. Multi-objective Optimization Problems (MOPs) involve multiple performance criteria or objectives which need to be optimized simultaneously (Fonseca and Fleming, 1995). Generally MOP can be formally defined as follows:

$$\begin{cases} \text{Maximize/Minimize: } F(x) = (f_1(x), f_2(x), \ldots, f_n(x)) \\ g_i(x) \le 0, \quad i = 1, \ldots, m \\ \text{and} \\ h_j(x) = 0, \quad j = 1, \ldots, q \end{cases} \tag{3.6}$$

where n is the number of objectives, g_i is the number of inequality constraints and h_j is the number of equality constraints. So, there are n objective functions considered in Eq. 3.6 and each function can be either minimized or maximized.

In the context of optimization, the duality principle (Deb, 2012) suggests that a maximization problem can be converted into a minimization one by multiplying the objective function with -1. This principle has made the optimization problems with mixed type of objectives easy to handle by transforming the objective into one same type of optimization problems (Lwin, 2015).

3.5.1. Techniques to solve multi-objective optimization problems

Several approaches have been developed in order to solve MOPs, they can be classified into three categories (Diwekar, 2008): the first one consists on methods based on converting the MOP into a single objective problem. Such approaches are known as priori approach, among them there are aggregation methods, ε-constraint and goal programming. The second category includes non-Pareto methods. They use operators that optimize each criterion independently. These approaches enabling the decision maker to intervene during the course of the resolution process are called interactive. The last one contains methods known as Pareto ones use the Pareto dominance concept for the selection of generated solutions. In the present thesis, we will focus on the first category of resolution by using an aggregation method and ε-constraints method.

3.5.1.1. Weighted sum method

The classical approach to solve a multi-objective optimization problem is to assign a weight ω_i to each objective function (Caramia and Olmo, 2008). So the problem is converted to a single objective problem with a scalar objective function as follows:

$$\text{Maximize/Minimize: } F(x) = \sum_{i=1}^{n} \beta_i \cdot f_i(x) \tag{3.7}$$

where, the weights β_i must be more than zero and their sum equal to one. The weighted sum method is the most used to solve the multi-optimization problems. However, the main disadvantage of this approach is that it requires a priori knowledge about the relative importance of the objectives (Konak et al, 2006).

3.5.1.2. ε-constraints Method

Besides the weighted sum method, another solution technique to multi-objective optimization is the ε-constraints method, which was originally proposed by Chankong and Haimes (1983). In this approach the decision maker chooses one objective out of n to be minimized; the remaining objectives are constrained to be less than or equal to given target values. Mathematically, if we let $f_2(x)$ be the objective function chosen to be minimized or maximized, we have the following problem:

$$\begin{aligned} &\text{Maximize/Minimize: } f_2(x) \\ &\text{Subject to: } f_i(x) \le \varepsilon_i, \forall \mathrm{i} \in \{1, \ldots, n\} \end{aligned} \tag{3.8}$$

where ε is the target value, which depending the specific characteristics of each problem.

3.6. Overview of differential evolution algorithm

The DE algorithm is very popular EAs, which was originally introduced by Storn and Price (1995) to solve the ***Chebyshev*** polynomial fitting problem. Similar to other evolutionary algorithms, DE is a population based optimization algorithm. The offspring compete with their parents inside the population and the winner will be kept as parents for next generation. The DE exhibits excellent capability in solving a wide range of optimization problems with different characteristics from several fields and many real-world applications.

Furthermore, there are a number of significant advantages when using DE (Price, 1999). Firstly, the DE is able to find the true global minimum/maximum regardless of the initial parameter values. The DE algorithm is fast and simple with regard to application and modification. In addition, the DE requires few control parameters comparing with the other EAs (such as GA). The recent versions of DE are effective to solve the nonlinear problems with the integer, discrete and mixed parameter optimization.

3.6.1. Basics of differential evolution algorithm

The DE technique involves three general evolutionary operators, i.e. mutation, crossover and selection, which are associated with certain control parameters. The procedure of the DE standard works as is given in Algorithm 3.2 (Mezura-Montes et al., 2006).

3.6.1.1. Parameters setting

Before the population can be initialized the control parameters of DE must be specified, which are: the population size NP, the maximum number of generations G_{max} (termination criterion), the scale factor F and the crossover rate Cr. An individual x with D decision variables represents a potential solution for the optimization problem (Lwin, 2015):

$$x = \{x_1, x_1, \ldots, x_D\}$$
$$L_i \le x_i \le U_i \tag{3.9}$$

where L_i and U_i are the lower and upper boundary constraints of the i^{th} variable respectively. At each generation G, DE maintains a population of NP individuals, $x_{i,j}^{G}, j \in [1, \ldots, NP]$.

Algorithm 3.1: Standard DE

Input:

NP: the number of population

G_{max}: the number of generation,

D: the number of dimension,

F: the scaling factor,

Cr: the crossover probability,

Output: x_{opt} the best individual with the smallest objective function value in the population

$G := 0$

Generate an initial population: $x_{i,j}^{G=0}, \forall j, j = 1, \ldots, NP$

Evaluate fitness of all initial individuals: $f\left(x_{i,j}^{G=0}\right), \forall j, j = 1, \ldots, NP$

for $G := 1$ to G_{max} **do**

  for $j := 1$to NP **do**

    Select randomly three integer indices $r_1, r_2, r_3 \in [1, NP] \wedge r_1 \neq r_2 \neq r_3 \neq j$

    Randomly generate an integer $i_{rand} \in \{1, \ldots, D\}$

    for $i :=$ to D **do**

      if $(rand(0,1) \le Cr)or\ (i == i_{rand})$ **then**

        $u_{i,j}^{G+1} := x_{i,r1}^{G} + F(x_{i,r2}^{G} - x_{i,r3}^{G})$

      else

        $u_{i,j}^{G+1} := x_{i,j}^{G+1}$

      end

    end

    if $f\left(u_{i,j}^{G+1}\right) \le f\left(x_{i,j}^{G}\right)$ **then**, for minimization case

      $x_{i,j}^{G+1} := u_{i,j}^{G+1}$

    else

      $x_{i,j}^{G+1} := x_{i,j}^{G}$

    end

  end

  $G := G + 1$

end

3.6.1.2. Initialization

The population *NP* is initialized by randomly generated individuals within the boundary constraints of each variable then the fitness of each individual will be evaluate.

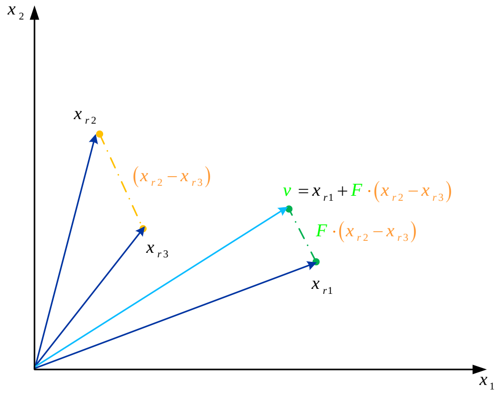
$$x_{i,j}^{G=0} = x_i^{(L)} + rand_{i,j}[0,1] \times \left(x_i^{(U)} - x_i^{(L)}\right) \tag{3.10}$$

where $rand_{i,j}[0,1]$ denotes a uniformly distributed random real value within the range [0, 1].

3.6.1.3. Mutation

DE mutation operation utilizes the differences of vector formed from chromosomes in the evolving population in order to determine both the degree and direction of perturbation applied to the mutant individual. The Fig. 3.1 illustrates how the mutant vector v is obtained in a two dimensional parametric space.

Fig. 3.1 Illustration of a basic DE mutation: the weighted differential, $F \cdot (x_{r2} - x_{r3})$ is added to the based vector x_{r1} to produce a trial vector v (Lwin, 2015)



After the initialization procedure, DE employs the mutation operation to produce a mutant vector. For each target vector $x_{i,j}^{G}$ a mutant vector $v_{i,j}^{G+1}$ is generated. The first variant of DE is DE/rand/1/bin-version:

$$v_{i,j}^{G+1} = x_{i,r1}^{G} + F(x_{i,r2}^{G} - x_{i,r3}^{G}) \tag{3.11}$$

Currently, a wide variety of strategies are developed for different purposes in literature. The most frequently used mutation strategies are (Qing, 2009):

- DE/best/1/bin:

$$v_{i,j}^{G+1} = x_{i,best}^{G} + F(x_{i,r1}^{G} - x_{i,r2}^{G}) \tag{3.12}$$

- DE/best/2/bin:

$$v_{i,j}^{G+1} = x_{i,best}^{G} + F_1(x_{i,r1}^{G} - x_{i,r2}^{G}) + F_2(x_{i,r3}^{G} - x_{i,r4}^{G}) \tag{3.13}$$

- DE/rand/2/bin:

$$v_{i,j}^{G+1} = x_{i,r1}^{G} + F_1(x_{i,r2}^{G} - x_{i,r3}^{G}) + F_2(x_{i,r4}^{G} - x_{i,r5}^{G}) \quad (3.14)$$

where $j = 1,2,\ldots, NP$. r_1, r_2, r_3, r_4, r_5 are the integer indices chosen randomly in the interval $[1,2,\ldots,NP]$ and $r_1 \neq r_2 \neq r_3 \neq r_4 \neq r_5 \neq j$. F, F_1 and F_2 are the scaling factors used to control the amplification of the differential variation between two individuals.

3.6.1.4. Crossover

Crossover is essential part of every EA methods, which is responsible for recombination of current with mutant population to produce trial population. There are two popular types of crossover methods used with the DE algorithm, namely exponential (or two-point modulo) and binomial (or uniform) (Das and Suganthan, 2011). In the binomial crossover the target vector is mixed with the mutated vector to yield the trial vector. The trial vector $u_{i,j}^{G+1}$ is generated using the target and mutated vectors as:

$$u_{i,j}^{G+1} = \begin{cases} v_{i,j}^{G+1}, \ if \ (rand_{i,j}[0,1] \leq Cr) \ or \ (i == i_{rand}) \\ x_{i,j}^{G}, \ otherwise \end{cases} \quad (3.15)$$

where i_{rand} is an integer index randomly chosen in the interval $[1,\ldots,D]$. The operator $(rand_{i,j})$ creates a random value uniformly distributed in the interval [0,1].

For an exponential crossover (Das et al., 2016), an integer n randomly chosen from $[1,\ldots,D]$. This integer acts as a starting point in the target vector, from where the crossover or exchange of components with the donor vector starts. In addition, another integer L is determined from $[1,\ldots,D]$. L denotes the number of components the donor vector actually contributes to the target vector. The pseudo-code of the exponential crossover is presented in the Algorithm 3.2:

Algorithm 3.2: Exponential crossover

```
L := 0
u_{i,j}^{G+1} := x_{i,j}^{G+1}
Randomly generate an integer j ∈ {1, ..., D}
while (rand[0,1] < Cr) and (L < d) do
    j := (j + 1) mod D
    v_{i,j}^{G+1} := x_{i,j}^{G+1}
    L := L + 1
end
```

3.6.1.5. Handling boundary constraints

As it's known, the recombination operation of DE is able to extend the search outside of the initialized range of the search space (Onwubolu and Davendra, 2006). In such case, the simple way is to replace those indexes that violated the boundary constraints with random values generated within the feasible range as follows:

$$u_{i,j}^{G+1} = \begin{cases} x_i^{(L)} + rand_{i,j}[0,1] \times \left(x_i^{(U)} - x_i^{(L)}\right), \; if \; \left(u_{i,j}^{G+1} < x_i^{(L)}\right) or \left(u_{i,j}^{G+1} > x_i^{(U)}\right) \\ u_{i,j}^{G+1}, \; otherwise \end{cases} \tag{3.16}$$

Another simple method that allows bounds to be approached asymptotically while minimizing the amount of disruption that results from resetting out of bound values (Price, 1999)

$$u_{i,j}^{G+1} = \begin{cases} \left(x_{i,j}^{G} + x_i^{(L)}\right)/2, \; if \; u_{i,j}^{G+1} < x_i^{(L)} \\ \left(x_{i,j}^{G} + x_i^{(U)}\right)/2, \; if \; u_{i,j}^{G+1} > x_i^{(U)} \\ u_{i,j}^{G+1}, \; otherwise \end{cases} \tag{3.17}$$

3.6.1.6. Selection

During the selection step, each individual from the candidate population is compared with its counterpart in the current population. The better one from the target vector $x_{i,j}^{G}$ and the trial vector $u_{i,j}^{G+1}$ will be chosen to enter the next generation according to their fitness value.

$$x_{i,j}^{G+1} = \begin{cases} u_{i,j}^{G+1} \; if \quad f\left(u_{i,j}^{G+1}\right) \leq f\left(x_{i,j}^{G}\right) \\ x_{i,j}^{G} \quad otherwise \end{cases} \tag{3.18}$$

3.6.1.7. Stopping criterion

If the stopping criterion is satisfied, the best result will be displayed. Otherwise, steps of mutation, crossover and selection are repeated. Six termination conditions are given in (Price et al., 2005), among them: objective met, limit the number of generations, population statistics, limited time, human monitoring and application specific. Bellow, we will expose briefly the most conditions used by the DE algorithm (Qing, 2009).

A. Limit the number of generations

Habitually, in the most cases of the optimization the objective function minimum value is not known in advance. Even for many test functions, only the best-known results are reported (Price et al., 2005). So, a stopping rule for problems with unknown optimum that is widely used in the literature is to terminate the execution of an algorithm after a given maximum number of function evaluations FEs or after G_{max} generations ($FEs = G_{max} \times NP$) (Ho-Huu et al., 2016; Li et al., 2016; Wang et al., 2016; Zielinski and Laur, 2008).

A. Objective met

For some optimization problems the objective function's minimum or maximum value is known in advance (test functions). So, an error measure depending on the difference to the optimum can be employed for detecting convergence of the algorithm (Zielinski et al., 2006). For more specification, the condition can be formulated as follows:

$$|fit\left(x_b^G\right) - f^*| \leq \epsilon \tag{3.19}$$

where $fit(x_b^G)$ denotes the fitness value of the best individual in the G^{th} generation, f^* denotes the known global optimal value, and ϵ is the specified tolerance of the global minimum, then the optimization halts (Zhang and Li, 2014).

B. Population statistics

An optimization can also be terminated when a population statistic reaches a pre-specified value. For example, an optimization can be halted when the difference between the population's worst and best objective function values falls below some predetermined limit. This method needs to be applied with caution because it can cause an optimization to halt prematurely. For example, if the optimization is halted when the difference between the population's worst and best objective function values is less than, e.g., 1.0×10^{-6}, the population's best objective function value might not yet be within 1.0×10^{-6} of the minimum value. Thus, the interruption is premature because DE may still be making progress even though the range of objective function values is small (Price et al., 2005).

3.6.2. Improving the performance of DE algorithm

DE is an optimization algorithm that has been extremely successful since its appearance. Originally, has been developed to deal with continuous and unconstrained optimization problems. Its current extensions could resolve problems with mixed variables and handle nonlinear constraints. However, like other evolutionary algorithms the DE's performance is mainly depends on two components. One is its trial vector generation strategy (i.e., mutation and crossover operators), and the other is its control parameters (i.e., population size *NP*, scaling factor *F*, and crossover probability *Cr*) (Wang et al., 2011). Generally, in the original DE the two control parameters *F* and *Cr* (Trivedi et al., 2017) are manually tuned according to the problem. Nevertheless, it is often time consuming to select a suitable trial vector generation strategy and tune its corresponding control parameters.

To improve the DE performance, several variants have been developed in recent decades. Some researchers are focused on the trial vector generation strategy. Indeed, Fan and Lampinen (2003) proposed a trigonometric mutation operator in order to accelerate the DE convergence. To implement the scheme, for each target vector, three distinct vectors are randomly selected from the DE population. Dass et al. (2005) introduced two schemes to adapt the scaling factor F in DE. The first scheme varies F in a random manner and the other one linearly reduces the value of F from a preset maximal value to a minimal one.

Further, Zhang and Sanderson (2009) proposed a new mutation strategy named "current-to-pbest/1". Instead of only adopting the best individual in the "current-to-best/1" strategy, their strategy also utilizes the information of other good solutions. In Das et al. (2009) the "current-to-best/1" strategy is improved by introducing a local neighborhood model. The new strategy muted each vector by using the best individual solution found so far in its small neighborhood. Wang et al. (2011) developed a new method called composite DE (CoDE), it combines three trial vector generation strategies "rand/1/bin, rand/2/bin and current-to-rand/1" with three values of F and Cr ($F =$ [1 1 0.8] and $Cr =$ [0.1 0.9 0.2]) they are chosen randomly.

Other research has focused on optimizing the DE control parameters (NP, F, Cr). For instance, Storn and Price (1995) suggested that NP should be between 5D and 10D, F should 0.5 as a good initial choice and the value of F smaller than 0.4 or larger than 1.0 will lead to performance degradation and Cr can be set to 0.1 or 0.9. Moreover, Gämperle et al. (2002) showed that the performance of DE is very sensitive to the setting of the control parameters based on their experiments on Sphere, Rosenbrock, and Rastrigin functions. They suggested that NP should be between 3D and 8D and the scaling factor F = 0.6, and the crossover rate Cr is between [0.3, 0.9]. Ronkkonen et al. (2005) suggested that NP should be taken in the range [2D, 4D]. They also recommended that Cr should be in [0, 0.2] when the function is separable, while in [0.9, 1] when the function's parameters are dependent and F should be chosen in the interval [0.4, 0.95]. For more detail there is survey review focused on DE and its recent advances, we refer the interested readers to the following references (Das and Suganthan, 2011; Das et al., 2016; Neri and Tirronen, 2010).

3.6.3. DE algorithm applications

As far as we know, with the advantages of simple structure, easy implementation and good computational efficiency, DE algorithm has been successfully applied to solve a wide range of optimization problems in wide variety of engineering applications. Qing (2009) demonstrated different applications of the DE in acoustics (Ganchev et al., 2004; Witos et al., 2006), chemical science and engineering (Angira and Babu 2006; Dragoi and Curteanu, 2016), gas oil and petroleum industry

(Nobakhti and Wang, 2006), defense (Starkey et al., 2005), etc. Furthermore, the DE algorithm has been also applied to solve signal processing (Lobato et al., 2012), pattern recognition (Maulik, 2009) and parameter estimation (Jiang et al., 2013). For more specification, a list of various scientific and commercial applications of DE is accessible online through the listed URLs. As the field of DE is very dynamic and rapidly changing, the list is updated very frequently. The list is gathered by a simple Google search for the keyword "Differential Evolution".

Over the last decade, the DE algorithm has been attracting increasing attention for electrical power system engineering (Sum-Im, 2009). There have been many researches that applied different version of the DE algorithm for solving electrical power system problems such as power system transfer capability assessment (Wong and Dong, 2005), power system planning (Dong et al., 2006), distribution network reconfiguration problem (Chiou et al., 2005), design of a gas circuit breaker (Kim et al., 2007), economic power dispatch (Wang et al., 2007), etc.

The performance of the DE in solving the structural mixed optimization problems has been excellently confirmed through many different engineering problems. For instance, Wang et al. (2009) applied a novel DE for designing optimal truss structures subjected to constraints of displacements and stresses with continuous and discrete design variables. The obtained results by authors were compared with various classical and evolutionary optimization methods. They showed effectively the robustness of proposed DE. In the same way, Wu and Tseng (2010) employed an adaptive Multi-Population DE (MPDE) augmented with a penalty-based self-adaptive strategy called Adaptive Multi-Population DE (AMPDE) to solve truss optimization problems with constraints.

In Le-Anh et al. (2015) an adjusted DE algorithm (aDE) and a smoothed triangular plate element for static and frequency optimization of folded laminated composite plates is used. The objective function was to minimize the strain energy for static problems and to maximize the fundamental frequency for free vibration problems. Also, the fiber orientations were taken as design variables which are discrete integer values between -90° and 90°. Moreover, the reliability and effectiveness of aDE were verified by comparing its numerical results with those of other algorithms in literature such as GA and PSO.

Ho-Huu et al. (2015) proposed a new version of the DE with a new discrete variables handling technique for layout optimization of truss with discrete design variables and constraints imposed on displacements and stresses. The objective function of the optimization problem was minimum weight of the whole truss structures. Numerical results, found by the authors, demonstrate that the proposed algorithm can effectively attain the best optimum solutions with less iteration than other methods in the literature. The same author Ho-Huu et al. (2016) developed a novel DE called ReDE which based on the roulette wheel selection members for the mutation phase instead of random selection as in the conventional DE. The method was proposed to solve the shape and size optimization problems for truss structures with frequency constraints. The efficiency and reliability of the proposed method are demonstrated through five numerical examples, and the numerical results reveal that the ReDE algorithm outperforms many optimization methods in the literature.

In addition to the truss layout optimization the DE algorithm has been used to optimize cutting parameters in milling operations Yildiz (2013a). The author presented a new hybrid approach based on differential evolution algorithm and receptor editing property of immune system. The results obtained by the proposed approach for milling operations indicate that the hybrid approach is more effective to optimize the cutting parameters for milling operations than the compared method including hybrid particle swarm algorithm, immune algorithm, hybrid immune algorithm, feasible direction method and handbook recommendations. The same author Yildiz (2013b) in another work used a novel hybrid optimization algorithm called Hybrid Robust DE (HRDE) for the same problem as in (Yildiz, 2013a).

In Weihmann et al. (2012) the parallel planar manipulator is optimized by using a Modified DE (MDE). Furthermore, in the literature several DE variants have been proposed for solving the five well known constrained benchmark mechanical design problems including pressure vessel design, speed reducer, welded beam, spring and three bar truss design. For example, in Montes et al. (2007) a modified version of the differential evolution algorithm is presented to solve engineering design problems. For constraint handling, the superiority of feasible points method proposed by (Deb, 2000) was adopted. In Huang et al. (2007), an effective Co-evolutionary DE (CDE) is introduced. In this work, a special penalty function was designed to handle the constraints. Simulation results based on several benchmark functions and three well-known engineering design problems as well as comparisons with some existed methods demonstrate the effectiveness, efficiency and robustness of the proposed method.

Moreover, Zhang et al. (2008) mixed the dynamic stochastic ranking with the multimember DE the variant was named (DSS-MDE). The performance of the developed method was demonstrated by solving 13 common benchmark functions and four well-studied engineering design examples. In the same way, in Wang and Li (2010) an effective differential evolution with level comparison (DELC) is proposed for constrained engineering design problems. They applied the level comparison to convert the constrained optimization problem into an unconstrained one and using the DE algorithm to perform a global search over the solution space. The effectiveness and efficiency of the proposed algorithm were demonstrated based on both typical benchmarks and several engineering design problems. Liu et al. (2010) proposed a novel hybrid algorithm named PSO-DE to solve constrained numerical and engineering optimization problems.

In Melo and Carosio (2013) the DE with using five common mutation strategies is used to solve two design problems, among pressure vessel design and spring design. It was found that all mutation strategies were able to get very close to the best known solution of each problem after a relatively small number of function evaluations. Furthermore, the compared results showed that the DE algorithm is more effective than some of the recent and sophisticated meta-heuristic approaches.

An improved constrained DE, named rank-iMDDE, is proposed by Gong et al. (2014) where two improvements of the DE are presented. Firstly, to make the DE algorithm converge faster a ranking-based mutation operator was added. Additionally, an improved dynamic diversity mechanism was proposed to maintain either infeasible or feasible solutions in the population. The authors evaluated the performance of rank-iMDDE through twenty four benchmark functions, five constrained engineering benchmark problems and four constrained mechanical design problems including (step-cone pulley, hydrostatic thrust bearing, rolling element bearing and belleville spring).

Recently, Yi et al. (2016) developed a new algorithm based on epsilon feasible individuals and Local Search (εDELS). The effectiveness of the proposed εDE-LS algorithm was tested thought four real-world engineering design problems and a case study. Panda et al. (2016) optimized the weight of a spur gear set and stress analysis of optimum gear set using a novel DE algorithm and FEA. The weight of a spur gear was formulated as a constrained optimization problem using the same formulation reported in (Savsani et al., 2010). In addition, a total of six design variables corresponding to gear geometry and material property were considered in the study.

3.7. Conclusion

Trough the short introduction presented about the recent related work of DE algorithm, it may be concluded that is difficult to select the adequate DE learning strategy or the optimal values of the control parameters. So in order to overcome this drawback we proposed to combine between different strategies or to use self-adapting control parameters.

CHAPTER FOUR:

Proposed algorithms:

Presentation and validation

PROPOSED ALGORITHMS: PRESENTATION AND VALIDATION

4.1. Introduction

The principal target of this chapter is to present in detail the two variants of the DE algorithm developed in this thesis which are the AMDE and modified AMDE (MAMDE). Moreover, the performance of the proposed algorithms are numerically tested using twelve problems including six widely used constrained engineering benchmarks and six real constrained mechanical design problems. The optimization results obtained by the developed variants are compared with those of the literature using different algorithms. In the second part, the proposed methods are applied to solve the problem of the optimal geometry of cylindrical gears.

4.2. Adaptive mixed differential evolution

Particularly the AMDE algorithm has been developed to select the optimal geometry parameters of a cylindrical spur gear, namely the profile shift coefficient, radius of root curvature and normal pressure angle. The objectives are to equalize the maximum bending stresses and to balance the specific sliding coefficients at extremes of contact path (this problem is well explained in chapter 5).

In addition, the robustness and the effectiveness of the proposed method is demonstrated by solving three well known practical engineering problems of mixed discrete continuous types including pressure vessel, speed reducer and multiple clutch disc brake problem. The particular variant used in order to generate a mutant vector is the DE/rand/2/bin scheme. The trial vector is generated using the binomial crossover.

Moreover, in order to avoid the manual tuning of DE parameters the AMDE algorithm uses a self adaptive approach, proposed by Brest et al. (2006) to adapt the control parameters (see section 4.2.2). In order to keep the best individual found in each generation, the elitism is performed. Further, a modified exterior penalty is applied to handle the design constraints. The basic idea of this method is to transform a constrained optimization problem into an unconstrained one, by adding the penalty terms into the objective function to penalize the constraint violations. The pseudo code of the AMDE algorithm is given in Algorithm 4.1 (Abderazek et al. 2017). The details steps of the proposed method are explained in the following sections.

Algorithm 4.1: AMDE algorithm

Input:

NP: number of population
G_{max}: maximum number of generations
D: number of dimension
Specify the probabilities to adjust the control parameters: τ_1, τ_2 and τ_3
Specify the initial values of control parameters: F_1 , F_2 and Cr_1

Output: x_{opt} the best individual with the smallest objective function value in the population

$G := 0$
Generate an initial population $x_{i,j}^{G=0}$:
  for $i := 1$ to D **do**
    Continuous variables: $x_{i,j}^{G=0} := x_i^{(L)} + rand_{i,j}[0,1] \times (x_i^{(U)} - x_i^{(L)})$
    Discrete and integer variables: $x_{i,j}^{G=0} = INT\left(x_i^{(L)} + rand_{i,j}[0,1] \times (x_i^{(U)} - x_i^{(L)} + 1)\right)$
  end
Evaluate the initial population and perform the elitism to choose f_{best} and x_{best} of $G = 0$
for $G := 1$ to G_{max} **do**
  for $j := 1$ to NP **do**
    Select randomly $r_1, r_2, r_3, r_4, r_5 \in [1, \ldots, NP]$ and $r_1 \neq r_2 \neq r_3 \neq r_4 \neq r_5 \neq j$
    Adjust the parameters $F_{1,j}$, $F_{2,j}$ and Cr_j
    Generate the mutant vector $v_{i,j}^{G+1}$ using DE/rand/2 mutation strategy
    for $i := 1$ to D **do**
      Continuous variables: $v_{i,j}^{G+1} := x_{i,r1}^{G} + F_{1,j}\left(x_{i,r2}^{G} - x_{i,r3}^{G}\right) + F_{2,j}\left(x_{i,r4}^{G} - x_{i,r5}^{G}\right)$
      Discrete and integer variables: $v_{i,j}^{G+1} := INT\left(x_{i,r1}^{G} + F_{1,j}\left(x_{i,r2}^{G} - x_{i,r3}^{G}\right) + F_{2,j}\left(x_{i,r4}^{G} - x_{i,r5}^{G}\right)\right)$
    end
    Handle boundary constraint violations according Eq.4.7
    Binomial Crossover
    Randomly generate an integer $i_{rand} \in \{1, \ldots, D\}$
    for $i := 1$ to D do
      if $rand_{i,j}[0,1] \leq Cr_{j,G+1}$ or $i = i_{rand}$ **then**
        $u_{i,j}^{G+1} := v_{i,j}^{G+1}$
      else
        $u_{i,j}^{G+1} := x_{i,j}^{G}$
      end
    end
    Evaluate the trial vector $f\,(u_{i,j}^{G+1})$ using penalty function to handle constraints
    if $f\left(u_{i,j}^{G+1}\right) \leq f\left(x_{i,j}^{G}\right)$ for minimization case, **then**
      $x_{i,j}^{G+1} := u_{i,j}^{G+1}$
    else
      $x_{i,j}^{G+1} := x_{i,j}^{G}$
    end
  end
  Perform the elitism to choose f_{best} and x_{best} of G
  $G := G + 1$
end

4.2.1. Parameters setting and initialization

Before the population can be initialized the control parameters of AMDE must be specified, which are: NP, G_{max}, the probabilities to adjust the control parameters: τ_1, τ_2 , τ_3 and the initial values of control parameters: F_1, F_2 and Cr_1. The fitness of each crated individual will evaluate and the best one is performed by the elitism to choose f_{best} and x_{best} of $G = 0$. The continuous variables are created randomly according to the Eq. 3.10 (see chapter 3).

4.2.1.1. Discrete and integer variables handling

Firstly, we should mention that the idea of treating the discrete and the integer variables as indexes is already existed in the literature (Kim et al., 2007; Liao, 2010; Onwubolu and Davendra, 2009). The main idea to handle a discrete variable is to use the variable position index in the array of normalized values. For example, when the created index is 3 this is the third normalized value that will be taken. So, if there are *n* normalized values in the set, there are *n* index positions from 1 to *n*. For more detail, the principle of discrete variable handling is presented in the Fig. 4.1. The integer variable can be treated in a similar way as the discrete.

More specifically, instead of optimizing the values of the discrete and the integer variables directly (Fig. 4.1), we optimize the values of their indexes which are created as real values, and then they are transformed to the nearest integer by truncation. During the evaluation of the objective function and the constraints, the discrete values are used instead of their indexes. In general, the handling of the discrete and the integer variables are performed in two procedures: initialization and mutation. The indexes are created as follows:

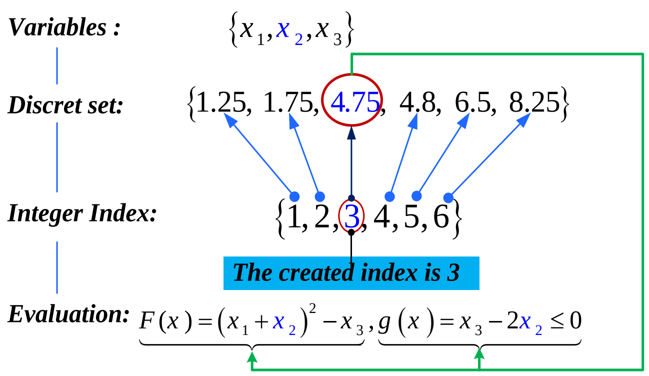
$$x_{i,j}^{G=0} = INT\left(x_i^{(L)} + rand_{i,j}\left[0,1\right] \times (x_i^{(U)} - x_i^{(L)} + 1)\right) \tag{4.1}$$

It is important to note that «1» is added to the Eq. 4.1 to ensure the upper limit of the discrete variable in the case the used *INT* function rounds the created value to the nearest integer less than or equal (for example the function floor of MATLAB). On another hand, if the *INT* function rounds the created value to the nearest integer (function round of MATLAB) the 1 is deleted and the indexes are created as follows:

$$x_{i,j}^{G=0} = INT\left(x_i^{(L)} + rand_{i,j}\left[0,1\right] \times (x_i^{(U)} - x_i^{(L)})\right) \tag{4.2}$$

where *INT* is a function that transforms the real value into the nearest integer value of $x_{i,j}^{G=0}$ by truncation. Each design variable $x_i, i = 1,\ldots,D$ is defined by a search space limited by lower L and upper U limits respectively.

Fig. 4.1 Discrete variable handling



4.2.2. Self adaptive approach (jDE)

In order to avoid the manual tuning of the DE parameters, the AMDE version used the self adaptive mechanism (jDE), proposed by Brest et al. (2006) to update the values of mutation and crossover factors. The jDE procedure is presented in Algorithm 4.2.

Algorithm 4.2: Self adaptive approach jDE

Specify the probabilities to adjust the control parameters: τ_1, τ_2 and τ_3
Specify the initial values of control parameters: F_1 , F_2 and Cr_1
Output: u

for $j := 1$ to NP **do**
  $\beta_1 := rand_{i,j}[0,1], \beta_2 := rand_{i,j}[0,1], \beta_3 := rand_{i,j}[0,1]$
  Adjust the parameters $F_{1,j}$, $F_{2,j}$ and Cr_j
  if $\beta_1 < \tau_1$ **then**
    $F_{1,j,G+1} := F_l + rand[0,1] \times F_u$
  else
    $F_{1,j,G+1} := F_{1,j,G}$
  end
  if $\beta_2 < \tau_2$ **then**
    $F_{2,j,G+1} := F_l + rand[0,1] \times F_u$
  else
    $F_{2,j,G+1} := F_{2,j,G}$
  end
  if $\beta_3 < \tau_3$ **then**
    $Cr_{j,G+1} := rand[0,1]$
  else
    $Cr_{j,G+1} := Cr_{j,G}$
  end
  Generate the mutant vector using DE/rand/2 mutation strategy
  Binomial Crossover
end

For each individual, the scaling factors $F_{1,j}$, $F_{2,j}$ and the crossover factor Cr_j are adjusted before the mutation is performed. So, they influence the mutation, the crossover and the selection operations of the new individual. The new control parameters are calculated as:

$$F_{1,j,G+1} = \begin{cases} F_l + rand\,[0,1] \times F_u & if \quad \beta_1 < \tau_1 \\ F_{1,j,G} & otherwise \end{cases} \tag{4.3}$$

$$F_{2,j,G+1} = \begin{cases} F_l + rand\,[0,1] \times F_u & if \quad \beta_2 < \tau_2 \\ F_{2,j,G} & otherwise \end{cases} \tag{4.4}$$

$$Cr_{j,G+1} = \begin{cases} rand\,[0,1] & if \quad \beta_3 < \tau_3 \\ Cr_{j,G} & otherwise \end{cases} \tag{4.5}$$

where $F_l = 0.1$, $F_u = 0.9$, β_k $(k = 1,\ldots,3)$ are uniform random values on [0, 1], $rand$ creates a random value uniformly distributed in the interval $[0,1]$. τ_1, τ_2, τ_3 represent the probabilities to adjust the control parameters.

4.2.3. Selection

In selection step, the better one from the target vector $x_{i,j}^{G}$ and the trial vector $u_{i,j}^{G+1}$ will be chosen to enter the next generation according to their fitness value.

$$x_{i,j}^{G+1} = \begin{cases} u_{i,j}^{G+1} & if \quad f\left(u_{i,j}^{G+1}\right) \le f\left(x_{i,j}^{G}\right) \\ x_{i,j}^{G} & otherwise \end{cases} \tag{4.6}$$

If the stopping criterion is satisfied, the best result will be displayed. Otherwise, steps of mutation, crossover and selection are repeated. Perform the elitism to choose f_{best} and x_{best} of G.

4.2.4. Constraints treatment

4.2.4.1. Handle boundary constraint violations

After using the DE mutation operator, for our case DE/rand/2 scheme, the i^{th} component of $v_{i,j}^{G+1}$ maybe is infeasible (i.e. violate the boundary constraint). In this work, we used the developed method of Wang et al. (2011) to deal with boundary constraint. Mathematically it can be formulated as follows:

$$v_{i,j}^{G+1} = \begin{cases} \min\left\{U_i, 2L_i - v_{i,j}^{G+1}\right\} & if \quad v_{i,j}^{G+1} < L_i \\ \max\left\{L_i, 2U_i - v_{i,j}^{G+1}\right\} & if \quad v_{i,j}^{G+1} > U_i \end{cases} \tag{4.7}$$

4.2.4.2. Constraints functions

The exterior penalty method has been widely used to deal with constraints, due to its simple principle and notably easy implementation (Gandomi et al. 2011; Ma et al. 2015; Zou et al. 2011; Zhang and Li, 2014). However, deciding optimal values of penalty terms especially for the optimization problems with highly constraints turns out to be a difficult optimization problem itself (Huang et al., 2007; Runarsson and Yao, 2000).

In order to overcome the above limitation, we propose to transform the constraint violation sum to an inequality constraint (Eq. 4.8). Then, the penalty term will be introduced into the objective function to treat this constraint (Eq. 4.9).

$$\phi(x) = \sum_{i=1}^{m} \max\left(0, g_i(x)\right) - \upsilon \leq 0 \tag{4.8}$$

where υ is a very small positive number chosen in the range [10^{-6}, 10^{-20}] depending the number and the constraint functions complexity of the problem.

$$F(x) = f(x) + \lambda\left(\max\left(0, \phi(x)\right)^2\right) \tag{4.9}$$

The main advantage of this method is that only one penalty factor will be set to solve the problem with constraints. So, it can be said that we handle the constraints in a similar way as the ε-constrained method (Takahama and Sakai, 2006), however, instead of using the lexicographical ordering mechanism we have used penalty function approach to punish the infeasible individuals.

In order to deal with equality constraints, the dynamic mechanism originally proposed by Hamida and Schoenauer, (2002) is adopted. The tolerance value is decreased with respect to the current generation using the following expression:

$$\epsilon_j(G+1) = \frac{\epsilon_j(G)}{1.00195} \tag{4.10}$$

The initial $\epsilon(0)$ was set to 0.001 (Eskandar et al., 2012).

4.3. AMDE with feasibility rules

The second proposed variant, in the present thesis, is considered as an improved version of the AMDE, where the variant is named Modified AMDE (MAMDE). MAMDE is reinforced by two mechanisms, namely the feasibility rules and constraints normalization. In the first step, the MAMDE version used the superiority of feasible points proposed by Deb (2000) in the selection phase instead the conventional one to improve the search capability of the variant, where the Deb's approach is based on three selection scenarios when comparing between the target vector $x_{i,j}^{G}$ and the trial vector $u_{i,j}^{G+1}$. The sequence of each criterion is given as follows:

- When two solutions are in the feasible region, the one with the better fitness value is selected;
- If one solution is feasible and the other is infeasible, the feasible one is preferred;
- Among two infeasible solutions, the one with the small constraint violation value is chosen. For more detail the feasibility rules procedure is described in Algorithm 4.3.

Algorithm 4.3: Feasibility rules approach in selection step

Feasibility rules approach in selection step

Compare between $x_{i,j}^{G}$ and $u_{i,j}^{G+1}$

if $\phi\left(u_{i,j}^{G+1}\right)=\phi\left(x_{i,j}^{G}\right)=0$ **then**, Two solutions are in the feasible region
    if $f\left(u_{i,j}^{G+1}\right)<=f\left(x_{i,j}^{G}\right)$
        $x_{i,j}^{G+1}:=u_{i,j}^{G+1}$
    else
        $x_{i,j}^{G+1}:=x_{i,j}^{G}$
    end
else
    if $\phi\left(u_{i,j}^{G+1}\right)\neq 0\wedge\phi\left(x_{i,j}^{G}\right)=0$ **then**, One solution is feasible and the other is infeasible
        $x_{i,j}^{g+1}:=x_{i,j}^{g}$
    else
        if $\phi\left(u_{i,j}^{G+1}\right)\neq 0\wedge\phi\left(x_{i,j}^{G}\right)\neq 0$ **then**, Both solutions are infeasible
            if $\phi\left(u_{i,j}^{G+1}\right)<\phi\left(x_{i,j}^{G}\right)$
                $x_{i,j}^{G+1}:=u_{i,j}^{G+1}$
            else
                $x_{i,j}^{G+1}:=x_{i,j}^{G}$
            end
        end
    end
end

For the constraints normalization we can say that the MAMDE used the combination between the exterior penalty functions and the superiority of feasible points. In addition, to assign the same degree of importance to all constraints, the maximum value of the violation of each constraint, for the entire population, is used to normalize each violated constraint (Venkatraman and Yen, 2005). Thus, the new constraint violation of an infeasible individual takes a value in the interval [0,1]. First, we find the maximum violation of each constraint in the population by using the following equation:

$$c_{\max}^{j} = \max c_j(x) \tag{4.11}$$

$$c(x) = \frac{1}{m}\sum_{j=1}^{m}\frac{c_j(x)}{c_{\max}^{j}} \tag{4.12}$$

where c_j is the constraint violation of an individual x on the j^{th} constraint, and m is the total number of the constraints.

4.4. Experimental results

In the present section, the efficiency of the AMDE and MAMDE versions are evaluated by optimizing six well known practical engineering benchmarks and six real mechanical design problems. In addition, for each problem we independently run each algorithm 50 times to measure the quality of the results and the robustness of the proposed variants. The developed algorithms are coded and implemented in MATLAB and the optimization runs were executed on a PC i5 with a 2.2 GHz Intel Dual Core processor and 4 GB of RAM memory.

4.4.1. Parameter tuning

The parameters of jDE for both proposed variants are as follows: initialization process sets are $F_1 =$ 0.2, $F_2 =$ 0.3, $Cr =$0.4, $\tau_1 =$0.5, $\tau_2 =$0.8 and $\tau_3 =$0.5. In order to compare between the proposed versions the maximum number of Function Evaluations (FEs) and the population size are taken the same, their values are summarized in the Table 4.1 for the two sets of test problems.

Table 4.1 Main parameters of the AMDE and MAMDE algorithms for each problem

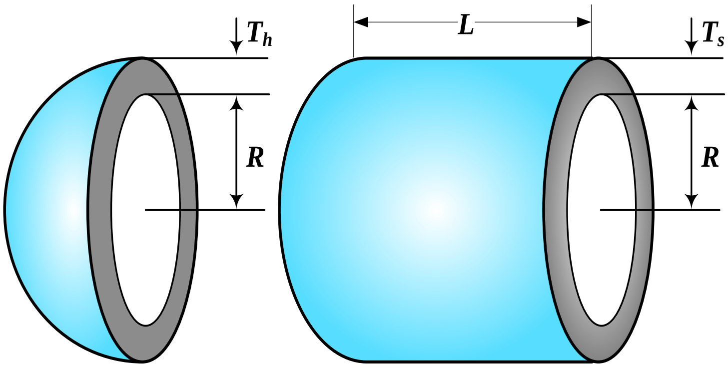
Problem	***n***	***NP***	***FEs***	**Problem**	***n***	***NP***	***FEs***
Pressure vessel	4	20	20,000	Multiple disc clutch brake	4	20	1,500
Speed reducer	7	20	25,000	Coupling with a bolted rim	4	25	10,000
Welded beam	4	30	18,000	Car side impact design	11	30	30,000
Tension/compression spring	3	35	20,000	Rolling element bearing	10	30	25,000
Three bar truss	2	20	4,000	Step-cone pulley	5	20	25,000
Stepped cantilever beam	10	65	150,000	Hydrodynamic thrust bearing	4	30	35,000

4.4.2. Constrained benchmark mechanical design problems

In this test stage, the effectiveness of each proposed algorithm is investigated on constrained engineering problems. For this reason, six well known constrained mechanical design problems have been chosen from the literature. They are the pressure vessel design, spring design, speed reducer design, welded beam design, three bar truss design and stepped cantilever beam design. The mathematical formulations of the benchmark engineering problems are given in details in Appendix A. Bellow the description of each problem is given.

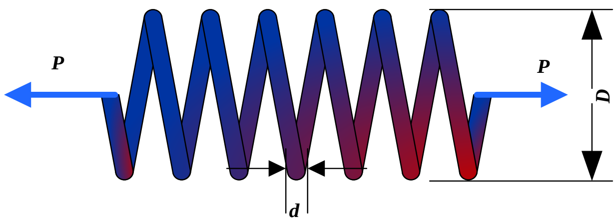
Pressure vessel: this problem was originally introduced by Sandgren (1990). The cylindrical vessel is capped at both ends by hemispherical heads, as shown in Fig. 4.2, must be designed for minimum total fabrication cost, including the material cost, forming and welding. There are four mixed design variables: the thickness of the cylindrical shell T_s, the thickness of the spherical head T_h, the inner radius R, and the length of the cylindrical segment of the vessel L. T_s and T_h are integer multiples of 0.0625 inch, R and L are continuous.

Fig. 4.2 Schematic view of cylindrical pressure vessel



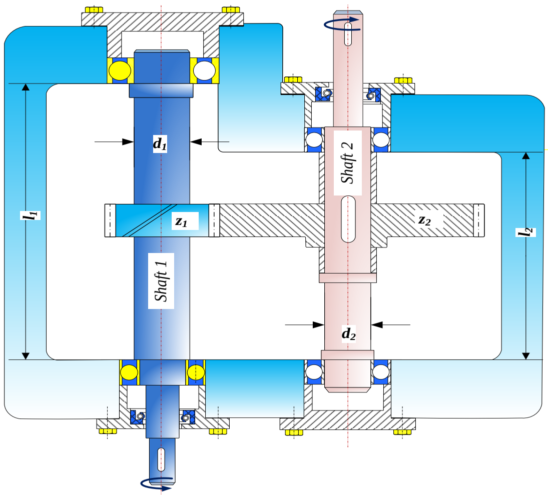
Tension/compression spring design problem: the second problem considered is minimizing the volume of a tension/compression spring (Arora, 2017). The spring is subjected to constraints on shear stress, surge frequency, and minimum deflection as shown in Fig. 4.3. There are three continuous design variables including the wire diameter d, the mean coil diameter D and the number of active coils N.

Fig. 4.3 Schematic view of tension/compression spring



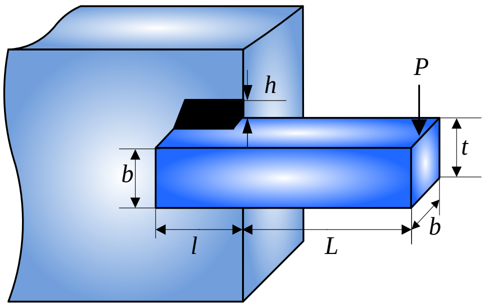
Speed reducer: the objective is to minimize the total weight of the speed reducer (Fig. 4.4) (Rao and Rao, 2009) subject to constraints on bending stress of the gear teeth, surfaces stress, transverse deflections of the shafts and stresses in the shafts. The variables are the face width b, module of teeth m, number of teeth on pinion z, length of first shaft between bearings l_1, length of second shaft between bearings l_2, diameter of shaft 1 d_1 and diameter of shaft 2 d_2.The variable z is integer and the rest of them are continuous.

Fig. 4.4 Schematic view of speed reducer



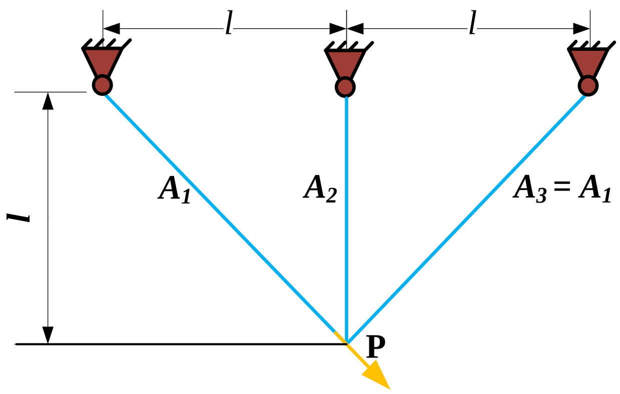
Welded beam design problem: the objective is to minimize the manufacturing cost of the welded beam (Rao and Rao, 2009) subject to constraints on shear stress τ, bending stress in the beam σ, buckling load on the bar Pc, end deflection of the beam δ and side constraints. There are four continuous variables (h, l, t and b) as shown in Fig.4.5.

Fig. 4.5 Schematic view of welded beam



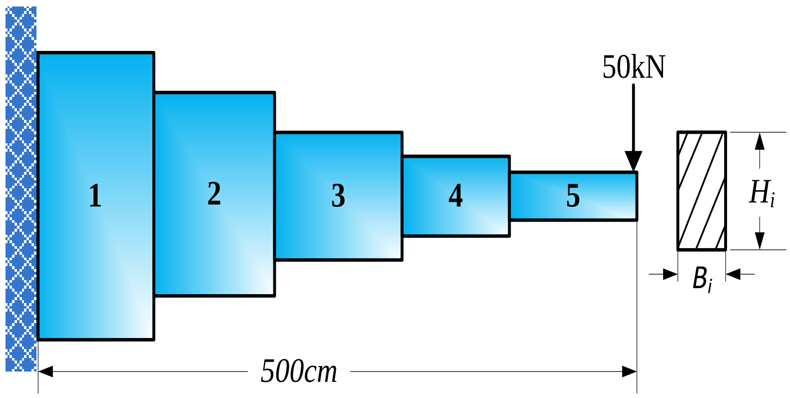
Three bar truss: this optimization problem was formulated for the first time by Nowcki (1973), where the volume of the three-bar truss must be minimized. The problem is subjected to stress constraints and the design variables are the cross sectional areas A_1 and A_2 as shown in Fig.4.6. The variables are continuous.

Fig. 4.6 Schematic view of three bar truss



Stepped cantilever beam design: another mixed optimization problem employed in this section which is the stepped cantilever beam design problem (Thanedar et al. 1995). The purpose is to minimize the volume of the five-cantilever beam subjected to a vertical load of 50 *kN* (Fig. 4.7). The height and width of the rectangular cross section of each step form the design variables. The constraints limit the deflection at the free end and the stresses in the beam. The geometry constraint is also applied to control the ratio of the width to the depth for each section. The variables B_1 and H_1 are integer, B_2, B_3, H_2 and H_3 are discrete, and B_4, H_4, B_5 and H_5 are continuous.

Fig. 4.7 Schematic view of stepped cantilever beam



4.4.2.1. Results and discussions

In the first step, we compared only the two developed methods in terms of the statistical results (to measure the robustness) and the best solution (to measure the quality of the solution) for each considered problem. The statistical results of all engineering design problems using AMDE and MAMDE are summarized in Table 4.2, in which the boldface indicates the best results among the both developed algorithms. In addition, the values of objective functions, design variables and constraints obtained by AMDE and MAMDE for each problem are presented in Tables 4.3 and 4.4 respectively. The convergence graph of the proposed algorithms are plotted in Figs. 4.8 to 4.13 for the pressure vessel, speed reducer, welded beam, spring design, three bar truss, and stepped cantilever beam, respectively.

From Tables 4.3 and 4.4, it can be said that the proposed algorithms are able to find the feasible global optimum solution for all engineering problems with very small standard deviation. In addition, the results are obtained with reasonably FEs number, no more than 150,000 in case of the stepped cantilever beam design problem. According to Table 4.2 and for the six problems, it is clear that MAMDE is more robust than the AMDE in terms of the standard deviation except for the problem of the three bar truss, where AMDE is slightly better than MAMDE. Besides, from the Figs. 4.8 to 4.13, it can be seen that our versions converged rapidly and can achieve the near-optimum within only 250, 200, 100, 70, 50 and 80 iterations for the pressure vessel, speed reducer, welded beam, spring design, three bar truss, and stepped cantilever beam respectively.

In addition, respect to the convergence speed it is observed that MAMDE is relatively faster than AMDE, except for the three bar truss where AMDE converged to the global solution before MAMDE. Moreover, for the stepped cantilever design we can't make the difference between the two methods. In accordance with above discussions it can be concluded that the proposed algorithms are very effective in solving the six engineering benchmarks in terms of solution quality, convergence speed and robustness.

Table 4.2 Statistical results obtained by AMDE and MAMDE for the six engineering design problems

Problem	Statistics	Algorithms	
		AMDE	MAMDE
Pressure vessel design problem	Best	6059.714335048435	6059.714335048435
	Mean	6059.714335048436	6059.714335048436
	Worst	6059.714335048443	6059.714335048436
	SD	1.4814E-12	**9.1872E-13**
Speed reducer design problem	Best	2994.471066146821	2994.471066146820
	Mean	2994.471066146938	2994.471066146820
	Worst	2994.471066148174	2994.471066146824
	SD	2.0702E-10	**4.1342E-12**
Welded beam design problem	Best	1.724852308597364	1.724852308597364
	Mean	1.724852308597365	1.724852308597365
	Worst	1.724852308597366	1.724852308597366
	SD	1.2342E-15	**9.3508E-16**
Tension/compression spring design problem	Best	0.012665232788319	0.012665232788319
	Mean	0.012665232788320	0.012665232788320
	Worst	0.012665232788335	0.012665232788323
	SD	2.5589E-15	**5.8876E-16**
Three bar truss design problem	Best	263.8958433764684	263.8958433764684
	Mean	263.8958433764685	263.8958433764685
	Worst	263.8958433764686	263.8958433764686
	SD	**2.6895E-13**	2.8188E-13
Stepped cantilever beam design problem	Best	64578.19402038647	64578.19401694651
	Mean	64578.19403421815	64578.19401698638
	Worst	64578.19416066698	64578.19401714868
	SD	2.0924E-05	**4.2055E-08**

Table 4.3 Values of objective functions, variables and constraints for engineering problems using AMDE

	Three bar truss design	Tension spring design	Pressure vessel design	Welded beam design	Speed reducer Design	Stepped cantilever design
Variables						
x_1	0.7886751330881	0.0516890608016	0.8125000000000	0.2057296397860	3.500000000000	3.00000000000
x_2	0.4082482947253	0.3567177330360	0.4375000000000	3.4704886656280	0.700000000000	60.0000000000
x_3	-	11.288966148130	42.098445595854	9.0366239103576	17.00000000000	3.10000000000
x_4	-	-	176.63659584243	0.2057296397860	7.300000000000	55.0000000000
x_5	-	-	-	-	7.715319911478	2.60000000000
x_6	-	-	-	-	3.350214666096	50.0000000000
x_7	-	-	-	-	5.286654464980	2.28088741670
x_8	-	-	-	-	-	45.6177483339
x_9	-	-	-	-	-	1.74975701194
x_{10}	-	-	-	-	-	34.9951402387
Constraints						
g_1	−1.0000E-13	−5.0000E-13	0	−1.0000E-12	−0.07391528	−3.5968E-8
g_2	−1.46410161	0	−0.03588082	0	−0.19799852	−1359.0500
g_3	−0.53589838	−4.05378561	0	0	−0.49917224	−153.84615
g_4	–	−0.72772880	−63.3634041	−3.43298420	−0.90464390	−1203.4124
g_5	–	–	–	−0.08072963	−2.5000E-12	−111.11111
g_6	–	–	–	−0.23554032	0	−2.6454E-12
g_7	–	–	–	−2.0000E-12	−0.70250000	−8.2966E-11
g_8	–	–	–	–	0	−2.0932E-11
g_9	–	–	–	–	−0.58333333	−0.76923076
g_{10}	–	–	–	–	−0.05132575	−2.25806451
g_{11}	–	–	–	–	0	0
Objective functions						
f	263.89584337646	0.0126652327883	6059.7143350484	1.7248523085973	2994.471066146	64578.1940170

Table 4.4 Values of objective functions, variables and constraints for engineering problems using MADME

	Three bar truss design	**Tension spring design**	**Pressure vessel design**	**Welded beam design**	**Speed reducer design**	**Stepped cantilever design**
			Variables			
x_1	0.7886751841137	0.0516890620837	0.8125000000000	0.2057296397860	3.50000000000	3.000000000000
x_2	0.4082481504031	0.3567177638808	0.4375000000000	3.4704886656280	0.70000000000	60.00000000000
x_3	–	11.288964339797	42.098445595854	9.0366239103576	17.0000000000	3.100000000000
x_4	–	–	176.63659584243	0.2057296397860	7.30000000000	55.00000000000
x_5	–	–	–	–	7.71531991147	2.600000000000
x_6	–	–	–	–	3.35021466609	50.00000000000
x_7	–	–	–	–	5.28665446498	2.280887416708
x_8	–	–	–	–	–	45.61774833386
x_9	–	–	–	–	–	1.749757011942
x_{10}	–	–	–	–	–	34.99514023875
			Constraints			
g_1	−1.2000E-13	−3.2000E-13	0	−1.0000E-12	−0.07391528	−2.7032E-8
g_2	−1.46410177	0	−0.03588082	0	−0.19799852	−1359.0500
g_3	−0.53589822	−4.05378567	0	0	−0.49917224	−153.84615
g_4	–	−0.72772878	−63.3634041	−3.43298420	−0.90464390	−1203.4124
g_5	–	–	–	−0.08072963	−8.1000E-13	−111.11111
g_6	–	–	–	−0.23554032	0	−8.0735E-13
g_7	–	–	–	−2.0000E-12	−0.70250000	−5.3962E-11
g_8	–	–	–	–	0	−1.2995E-10
g_9	–	–	–	–	−0.58333333	−0.76922307
g_{10}	–	–	–	–	−0.05132575	−2.25801290
g_{11}	–	–	–	–	0	0
			Objective functions			
f	263.89584337647	0.0126652327883	6059.7143350484	1.7248523085973	2994.47106614	64578.194016994

Fig. 4.8 Convergence graph of AMDE and MAMDE for pressure vessel design problem

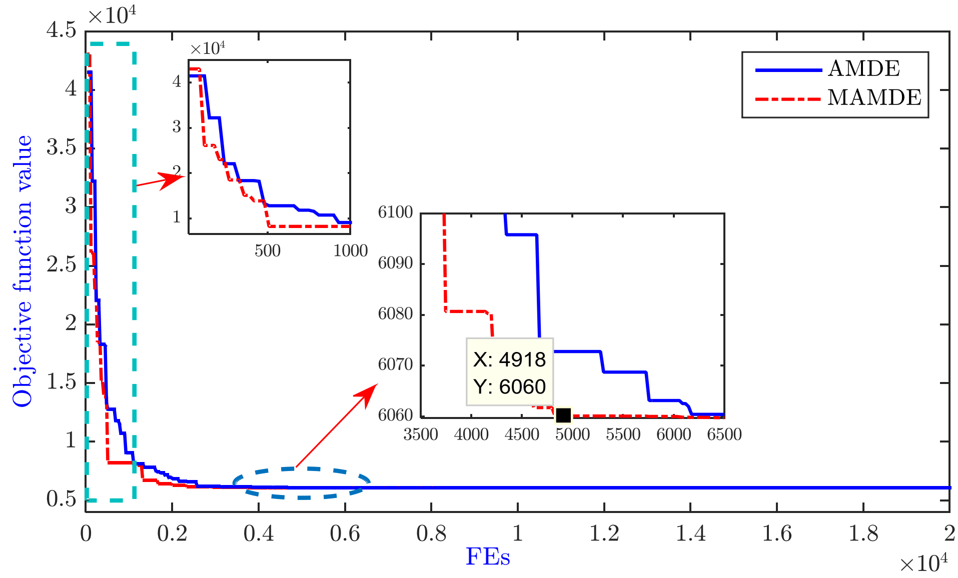


Fig. 4.9 Convergence graph of AMDE and MAMDE for speed reducer design problem

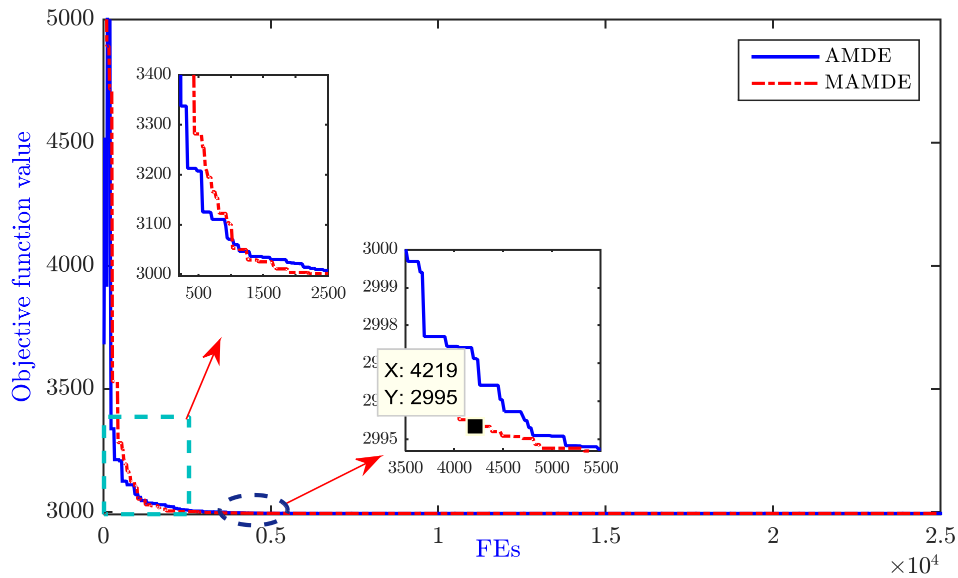


Fig. 4.10 Convergence graph of AMDE and MAMDE for welded beam design problem

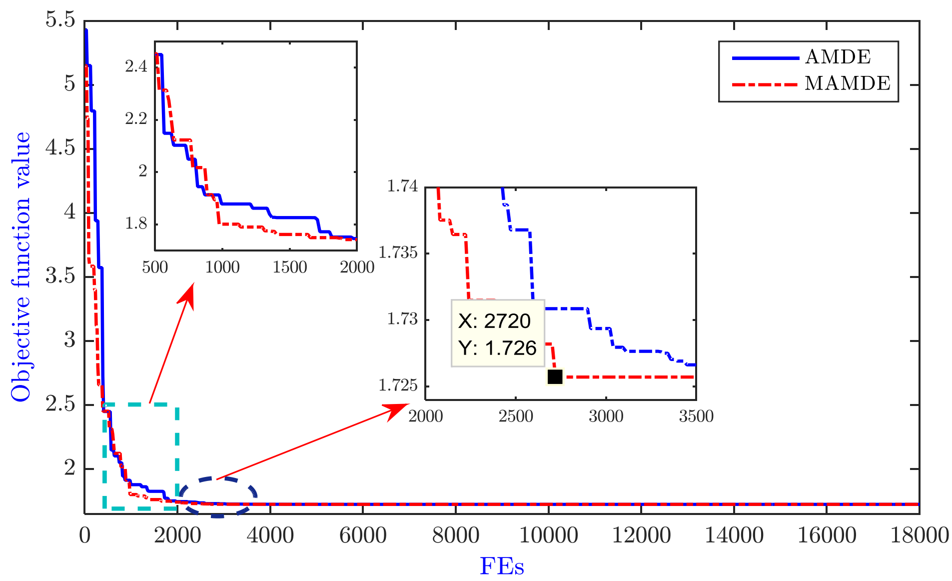


Fig. 4.11 Convergence graph of AMDE and MAMDE for spring design problem

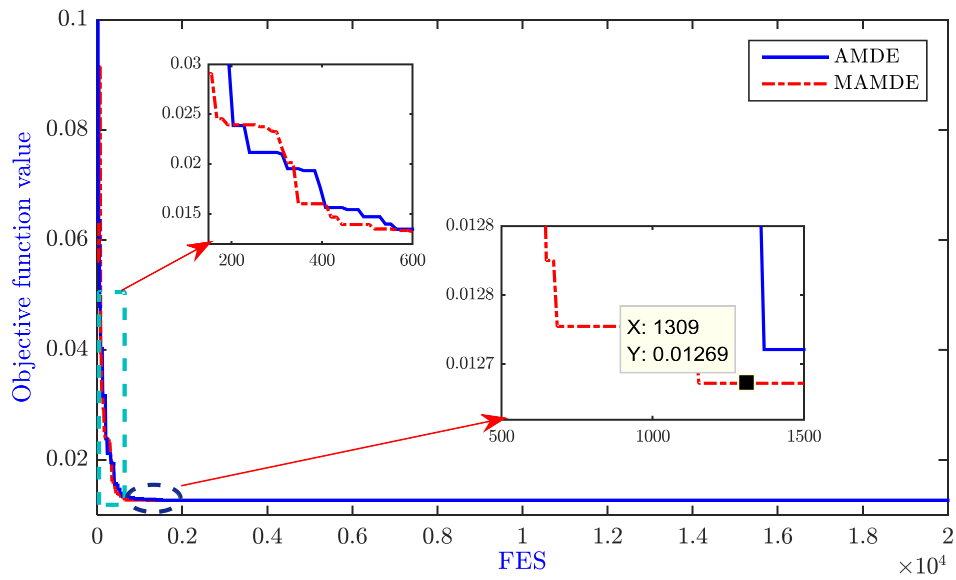


Fig. 4.12 Convergence graph of AMDE and MAMDE for three bar design problem

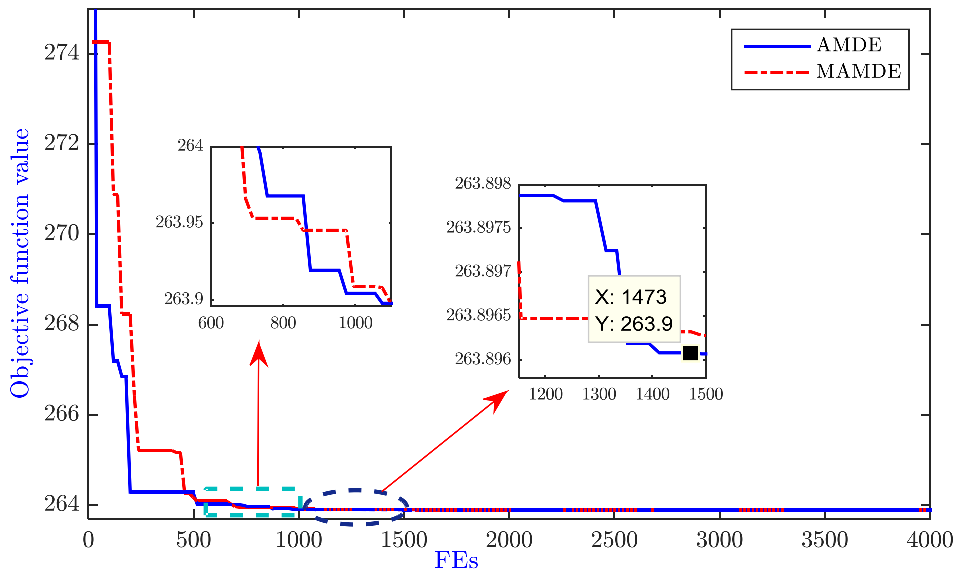
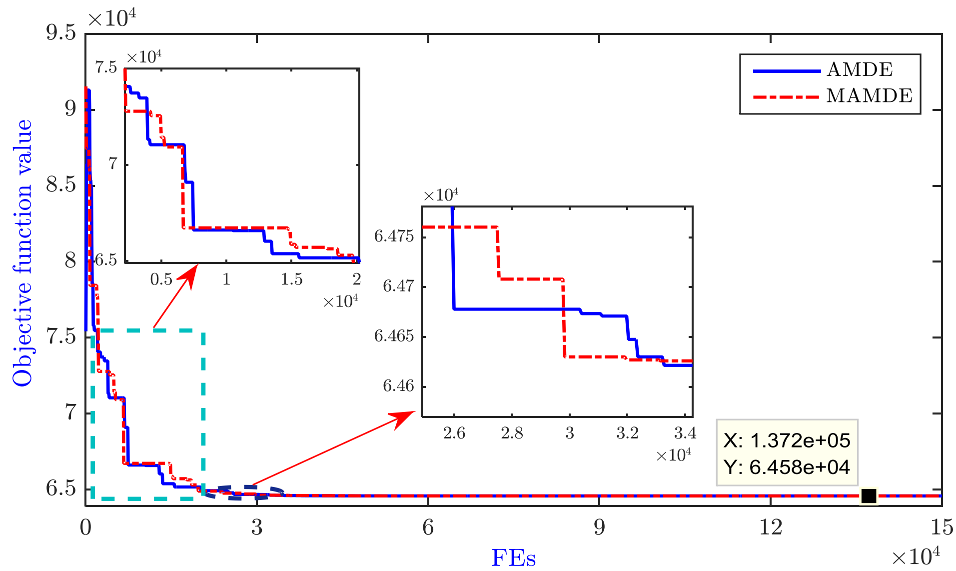


Fig. 4.13 Convergence graph of AMDE and MAMDE for stepped cantilever design problem



4.4.2.2. Comparison with other algorithms

Several methods are recently used in the literature to solve the six engineering problems, AMDE and MAMDE are compared with the following metaheuristics: modified Backtracking Search Optimization Algorithm (SSBSA) (Wang et al., 2018), ε-constrained DE algorithm with a novel Local Search operator (εDE-LS) (Yi et al., 2016a), pre-estimate comparison gradient based approximation ε-constrained differential evolution (εDE-PCGA) (Yi et al., 2016b), Rank-iMDDE (Gong et al., 2014), Social-Spider algorithm (SSO-C) (Cuevas and Cienfuegos, 2014), Upgraded Artificial Bee Colony (UABC) (Brajevic and Tuba, 2013), Constrained Optimization based on a Modified DE (COMDE) (Mohamed and Sabry, 2012), Water Cycle Algorithm (WCA) (Eskandar et al., 2012), DE with Level Comparison (DELC) (Wang and Li, 2010), hybrid PSO-DE (Liu et al., 2010), Accelerated Adaptive Trade-off Model based EA (AATM) (Wang et al., 2009), Dynamic Stochastic ranking based DE (DSS-MDE) (Zhang et al., 2008), and Multiple trial vector based DE (MDDE) (Mezura-Montes et al., 2007).

The selected compared algorithms for the five first engineering optimization problems have not been used in solving the stepped cantilever design problem. For this reason, we choose other algorithms for the comparison which are Particle Swarm assisted GA (PSGA) (Dhadwal et al., 2014), GA hybridized with the Artificial Immune System (AIS-GA) (Bernardino and Barbosa, 2008), GA with the Stochastic Ranking (SR) (Bernardino and Barbosa, 2008), GA with an Adaptive Penalty method (APM) (Lemonge and Barbosa, 2004), and GA based Optimum Structural design (GAOS) (Erbatur et al., 2000).

For the presser vessel design problem, the AMDE and the MAMDE are compared with eight algorithms including SSBSA, εDE-LS, εDE-PCGA, Rank-iMDDE, UABC, COMDE, DELC, and PSO-DE. The statistical results for the compared algorithms are given in Table 4.5. From the Table we can see that all algorithms are able to solve this problem by providing the global solution of 6059.714335. In terms of the standard deviation, εDE-LS and MAMDE outperform the other compared algorithms with values of 3.4030E-13 and 9.1872E-13, respectively. In addition, the statistical results provided by using the AMDE version is similar to those of the Rank-iMDDE. In terms of best solution it seems that SSBSA is the most successful algorithm. Nevertheless, the produced results are infeasible solution because the first constraint had been violated (Table 16 in Wang et al., 2018).

The speed reducer problem has been studied by several approaches in the literature, such as SSBSA, εDE-LS, εDE-PCGA, Rank-iMDDE, UABC, COMDE, DELC, and PSO-DE. The statistical results of the metaheuristics are listed in Table 4.6. From the Table, we can see that except to the PSO-DE, all algorithms are able to find the optimal solution for this problem. Further, Rank-iMDDE provides the smallest standard deviation. Among these methods MAMDE is slightly worse than

COMDE, DELC, εDE-LS regarding the robustness. The results given by AMDE are equal to those of εDE-PCGA, and better than those of SSBSA, UABC, and PSO-DE.

Table 4.5 Comparison with other state-of-the-art algorithms on pressure vessel design problem

Algorithm	Best	Mean	Worst	SD	FEs
MAMDE	6059.714335048435	6059.714335048436	6059.714335048436	**9.1872E-13**	20,000
AMDE	6059.714335048435	6059.714335048436	6059.714335048443	1.4814E-12	20,000
SSBSA	6059.708274	6464.939874	7273.509192	316.685100	32,640
εDE-LS	6059.7143	6059.7143	6059.7143	**3.4030E−13**	20,000
εDE-PCGA	6059.714335	6059.714335	6059.714335	3.96E−09	20,000
Rank-iMDDE	6059.714335	6059.714335	6059.714335	1.95E-12	23,465
UABC	6059.714335	6192.116211	NA	2.04E+02	15,00
COMDE	6059.714335	6059.714335	6059.714335	1.95E-10	30,000
DELC	6059.7143	6059.7143	6059.7143	2.10E-11	30,000
PSO-DE	6059.714335	6059.714335	6059.714335	1.0E−10	42,100

Table 4.6 Comparison with other state-of-the-art algorithms on speed reducer design problem

Algorithm	Best	Mean	Worst	SD	FEs
MAMDE	2994.471066146820	2994.471066146820	2994.471066146824	4.1342E-12	25,000
AMDE	2994.471066146821	2994.471066146938	2994.471066148174	2.0702E-10	25,000
SSBSA	2994.468370	2998.010116	3171.447162	25.028268	6465
εDE-LS	2994.471066	2994.471066	2994.471066	1.90E−12	30,000
εDE-PCGA	2994.4710661	2994.4710661	2994.4710661	1.16E−10	20,000
Rank-iMDDE	2994.471066	2994.471066	2994.471066	**7.93E−13**	19,920
UABC	2994.471066	2994.471072	NA	5.98E−06	15,000
COMDE	2994.4710661	2994.4710661	2994.4710661	1.54E−12	21,000
DELC	2994.471066	2994.471066	2994.471066	1.90E−12	30,000
PSO-DE	2996.348165	2996.348165	2996.348165	1.0E-07	54,350

For the welded beam design problem both developed algorithms are compared with eight methods. They are SSBSA, Rank-iMDDE, COMDE, DELC, WCA, MDDE, SSO-C and PSO-DE. The comparative results of these eight techniques and the proposed variants are given in Table 4.7. It can be observed from the Table that all compared algorithms are able to achieve the global optimal solution in all runs. With respect to the robustness, Rank-iMDDE and NAMDE outperform the other compared algorithms with standard deviation value of 9.06E-16 and 9.3508E-16, respectively, followed by AMDE, MDDE, DELC and COMDE. However, MAMDE and AMDE find the global optimum with less number of function evaluations compared with other algorithms. Accordingly, MAMDE is the most efficient among the reported algorithms in solving this problem.

Table 4.7 Comparison with other state-of-the-art algorithms on welded beam design problem

Algorithm	Best	Mean	Worst	SD	FEs
MAMDE	**1.724852308597364**	**1.724852308597365**	**1.724852308597366**	**9.3508E-16**	**18,000**
AMDE	1.724852308597364	1.724852308597365	1.724852308597366	1.2342E-15	**18,000**
SSBSA	1.724851	1.910178	2.217041	0.154774	34,340
Rank-iMDDE	1.724852309	1.724852309	1.724852309	**9.06E-16**	19,830
SSO-C	1.7248523085	1.746461619	1.799331766	2.57E−02	25,000
COMDE	1.724852	1.724852	1.724852	1.60E−12	20,000
DELC	1.724852	1.724852	1.724852	4.10E−13	20,000
WCA	1.724856	1.726427	1.744697	4.29E−03	46,450
PSO-DE	1.7248531	1.7248579	1.7248811	4.1E-06	33,000
MDDE	1.725	1.725	1.725	1.00E−15	24,000

The spring design problem has been solved by SSBSA, εDE-LS, εDE-PCGA, Rank-iMDDE, COMDE, WCA, DELC, DSS-MDE, and AATM. The results of these algorithms are shown in Table 4.8. As it can be seen from the table, best feasible solution obtained is 0.01266523 which provided by all compared EAs. Among these methods our variants obtain the best results in terms of best, mean and worst solutions. In addition, in terms of the standard deviation it is clearly observe that the MAMDE provide the small one with 5.8876E-16 followed by the AMDE with 2.5589E-15. Therefore, MAMDE algorithm is most robust in solving this problem.

Table 4.8 Comparison with other state-of-the-art algorithms on tension spring design problem

Algorithm	Best	Mean	Worst	SD	FEs
MAMDE	**0.012665232788319**	**0.012665232788320**	**0.012665232788323**	**5.8876E-16**	20,000
AMDE	**0.012665232788319**	**0.012665232788320**	**0.012665232788335**	2.5589E-15	20,000
SSBSA	0.012665	0.012749	0.013300	1.31E-04	36,030
εDE-LS	0.012665233	0.012665233	0.012665233	5.0075E−14	20,000
εDE-PCGA	0.012665233	0.012665233	0.012665235	4.43E−10	15,000
Rank-iMDDE	0.012665233	0.012665264	0.01266765	2.45E−07	19,565
UABC	0.012665	0.012683	NA	3.31E−05	12,000
COMDE	0.012665233	0.012665233	0.012665233	3.09E−06	24,000
WCA	0.012665	0.012746	00.012952	8.06E−05	11,750
DELC	0.012665233	0.012665267	0.012665575	1.30E−07	20,000
DSS-MDE	0.012665233	0.012669366	0.012738262	1.25E−05	24,000
AATM	0.012668262	0.012708075	0.012861375	4.50E−05	25,000

The three bar truss design problem has been recently solved using SSBSA, εDE-LS, εDE-PCGA, Rank-iMDDE, UABC, COMDE, WCA, DELC, and DSS-MDE. The results of these algorithms are given in Table 4.9. As it can be seen from the table, Rank-iMDDE and UABC give the best standard deviation value followed by εDE-LS, εDE-PCGA, DELC, NAMDE, and COMDE. However, AMDE and MAMDE show superiority in terms of number function evaluations (4,000).

Table 4.9 Comparison with other state-of-the-art algorithms on three bar truss problem

Algorithm	Best	Mean	Worst	SD	FEs
MAMDE	263.8958433764684	263.8958433764685	263.8958433764686	2.8188E-13	**4,000**
AMDE	263.8958433764684	263.8958433764685	263.8958433764686	2.6895E-13	**4,000**
SSBSA	263.895843	263.895844	263.895850	9.94E-07	6135
εDE-LS	263.895843376468	263.895843376468	263.895843376468	2.3206E−14	15,000
εDE-PCGA	263.89584	263.89584	263.89584	2.1268E−14	15,000
Rank-iMDDE	263.8958434 0	263.8958434	263.8958434	**0.00E+00**	4,920
UABC	263.895843	263.895843	NA	**0.00E+00**	12,000
COMDE	263.8958434	263.8958434	263.8958434	5.34E−13	7000
WCA	263.895843	263.895843	263.895843	8.71E−05	5,250
DELC	263.8958434	263.8958434	263.8958434	4.34E−14	10,000
DSS-MDE	263.8958433	263.8958436	263.8958498	9.70E−07	15,000

The stepped cantilever beam design problem presents a practical engineering optimization problem with mixed discrete and continuous design variables. This problem has been solved in the literature by using GAOS, APM, AIS-GA, SR, and PSGA. The results of the seven compared methods are presented Table 4.10. It can be clearly observed from the Table that MAMDE and MADE provide new solution for this problem (64578.19401). In addition, MAMDE and AMDE obtain the best results compared with algorithms in terms of best, mean and worst solutions. The worst values obtained by AMDE and MAMDE are better than the best solution obtained by DELC and PSGA. Thus, MAMDE algorithm is most effective in solving this problem.

Table 4.10 Comparison with other state-of-the-art algorithms on stepped cantilever beam problem

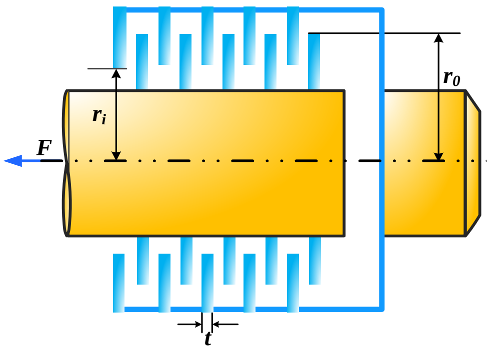
Algorithm	Best	Mean	Worst	SD	FEs
MAMDE	64578.19401694651	64578.19401698638	64578.19401714868	**4.2055E-08**	150,000
AMDE	64578.19402038647	64578.19403421815	64578.19416066698	**2.0924E-05**	150,000
PSGA	64578.374336	66667.003313	69495.390298	1.90E+03	**5,000**
SR	64599.65	83968.45	71240.03	3.90E+03	35,000
AIS-GA	64,834.70	76,004.24	102,981.06	6.93E+03	35,000
APM	64,698.56	6,8107.046	73931.359	NA	35,000
GAOS	64815	NA	NA	NA	10,000

4.4.3. Constrained mechanical design problems

To further verify the performance of the AMDE and MAMDE on solving real world constrained optimization problems, five mechanical design problems are considered, including the multiple clutch disc brake, coupling with a bolted rim, car side impact, rolling element bearing, step-cone pulley, and hydrodynamic thrust bearing. The maximization problems, in case of rolling element, are transformed into a minimization one by multiplying the objective function with -1. The equality constraints are converted into inequality ones, the initial tolerance value is set to 0.001. The details of the mathematical formulation of each problem are given in Appendix B. Moreover, bellow the description of each problem is given.

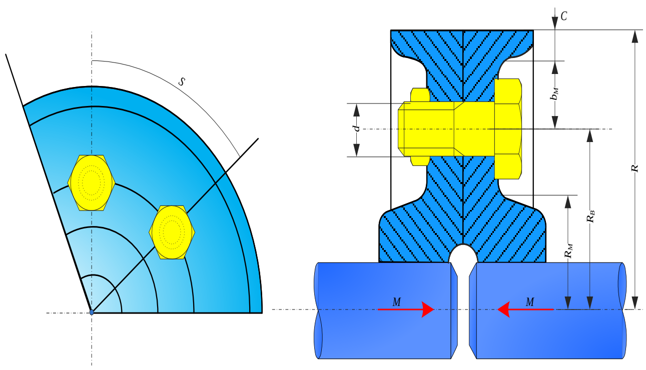
Multiple disc clutch brake: the multiple disc clutch brake (Rao and Rao, 2009), Fig. 4.14, must be designed for the minimum weight using five discrete variables: inner radius $r_i \in (60, 61, 62,\ldots, 80)$, outer radius $r_0 \in (90, 91,\ldots,110)$, thickness of discs $t \in (1, 1.5, 2, 2.5, 3)$, actuating force $F \in (600, 610, 620,\ldots, 1000)$ and number of friction surfaces $Z \in (2, 3, 4, 5, 6, 7, 8, 9)$.

Fig. 4.14 Schematic view of multiple disc clutch brake



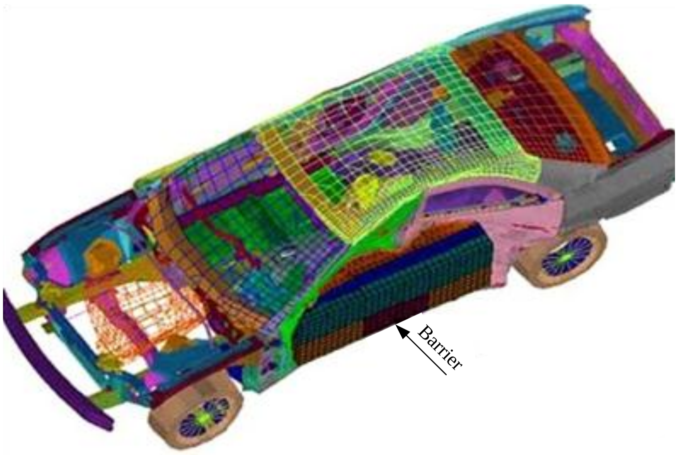
Coupling with a bolted rim: the coupling with a bolted rim problem was firstly proposed by Lafon (1994). As shown in Fig. 4.15, a torque M is transmitted by adhesion using N bolts of diameter d placed at radius R_B. The objective function includes three terms with weighting coefficients β_1, β_2, and β_3. d (x_1) is discrete, N (x_2) is integer, R_B (x_3) and M (x_4) are continuous variables. The problem is subjected to eleven inequality constraints.

Fig. 4.15 Schematic view of a coupling with bolted rim



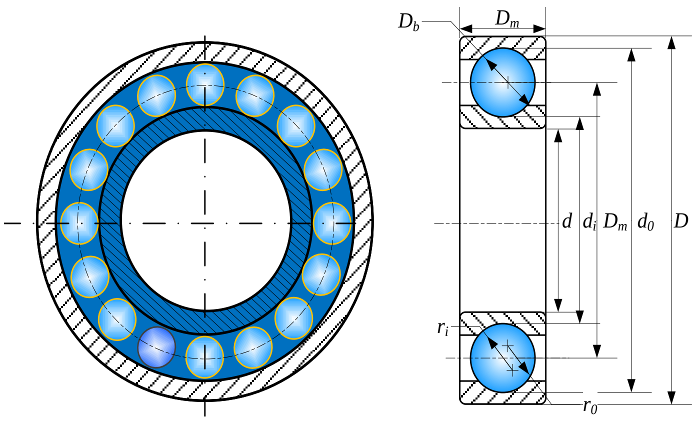
Car side impact design: the car side impact design was originally proposed by Gu et al. (2001). The car is exposed to a side impact on the foundation of the European Enhanced Vehicle Safty Committee (EEVC) procedures. The finite element model of this problem is illustrated in the Fig. 4.16. The objective function of the problem is to minimize the total weight of the car using eleven decision variables. The eight and the nine variable are integers and the rest of them are continuous. The problem is subjected to ten inequality constraints.

Fig. 4.16 Finite element model utilized in the car side impact problem (Youn and Choi, 2004)



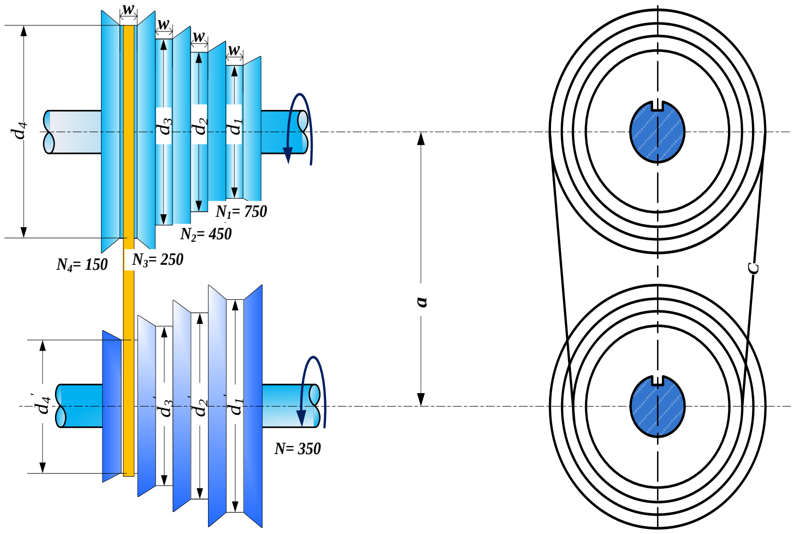
Rolling element bearing: the rolling element bearing must be designed for maximum dynamic load carrying capacity (Gupta et al. 2007).The design variables are pitch diameter D_m, ball diameter D_b, number of balls Z, inner and outer raceway curvature coefficients f_i and f_o (Fig. 4.17). Moreover, all design variables are continuous except the number of balls that is discrete. The problem is subjected to nine physical constraints.

Fig. 4.17 Schematic view of rolling element bearing



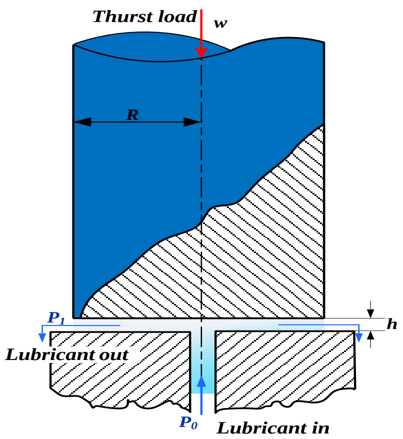
Step-cone pulley: the objective of this problem (Fig. 4.18) is to design a 4 step-cone pulley with minimum weight (Rao et al. 2011). The problem include five variables, the diameters of each step $d_i\,(x_i), (i = 1,\ldots,4)$, and the width of the pulley $\omega(x_5)$. A total of eleven constraints are considered, three equality constraints and eight inequality constraints. The step pulley transmitted at least 0.75 *hp*, with an input speed of 350 *rpm* and output speeds of 750, 450, 250 and 150 *rpm*. All variables are continuous.

Fig. 4.18 Schematic view of step-cone pulley



Hydrostatic thrust bearing: the problem was originally proposed by Coello (2000), where the objective is to minimize the power loss during the operation of a hydrostatic thrust bearing (Fig.4.19). There are four design variables including bearing step radius R, recess radius R_o, oil viscosity μ, and flow rate Q. Seven different constraints are associated with the problem, based on load carrying capacity, inlet oil pressure, oil temperature rise, and oil film thickness. The four variables are continuous.

Fig. 4.19 Schematic view of hydrodynamic thrust bearing



4.4.3.1. Results and discussions

For each mechanical engineering problem we compared the solutions obtained by AMDE and MAMDE, the results are provided including the best, mean and worst solutions and the standard deviation. The statistical results for the six mechanical engineering design problems using AMDE and

MAMDE are summarized in Table 4.11. Besides, the values of objective functions, design variables and constraints obtained by the two developed versions for each problem are tabulated in Table 4.12 and Table 4.13 respectively. The convergence graph of the proposed approaches for each mechanical problem are plotted in Figs. 4.20 to 4.25 for the multiple clutch disc brake, coupling with a bolted rim, car side impact design, rolling element bearing, step-cone pulley, and hydrodynamic thrust bearing, respectively.

From Tables 4.12 and 4.13, it is clear that the two developed algorithms are able to achieve the feasible global solutions for all mechanical engineering problems with small FEs number, no more than 35,000 in case of the hydrodynamic thrust bearing problem . Moreover, form the Table 4.11, the MAMDE is more robust than the AMDE in solving the multiple clutch disc brake, the coupling with a bolted rim, and the hydrodynamic thrust bearing. However, for the rest of problems the two algorithms are almost having the same standard deviation. For the convergence speed, it's clear from the Figs. 4.20 to 4.25 that the MAMDE version is relatively much faster than the AMDE for all mechanical problems, except for the hydrodynamic thrust bearing where AMDE converged to the best solution before MAMDE. Besides, from the figures it can be seen that our two algorithms converged rapidly and can achieve the near-optimum within only 40, 45, 50, 25, 250 and 60 iterations for the multiple clutch disc brake, coupling with a bolted rim, car side impact design, rolling element bearing, step-cone pulley, and hydrodynamic thrust bearing, respectively.

From the above discussions, it can be concluded that the AMDE and MAMDE variants are very effective in optimizing the six mechanical engineering design problems in terms of solution quality, convergence speed and robustness.

Fig. 4.20 Convergence graph of AMDE and MAMDE for multiple clutch disc design problem

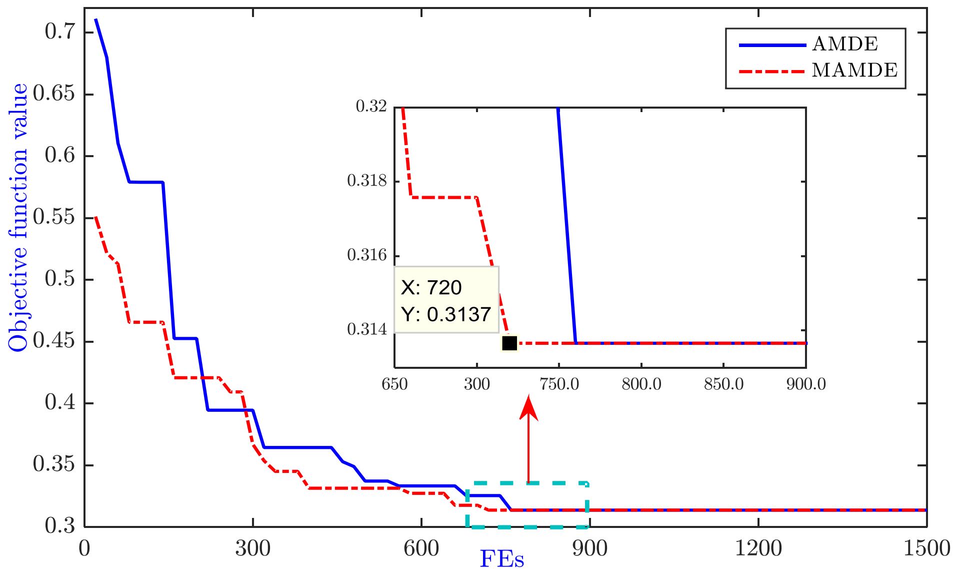


Fig. 4.21 Convergence graph of AMDE and MAMDE for coupling with a bolted rim design problem

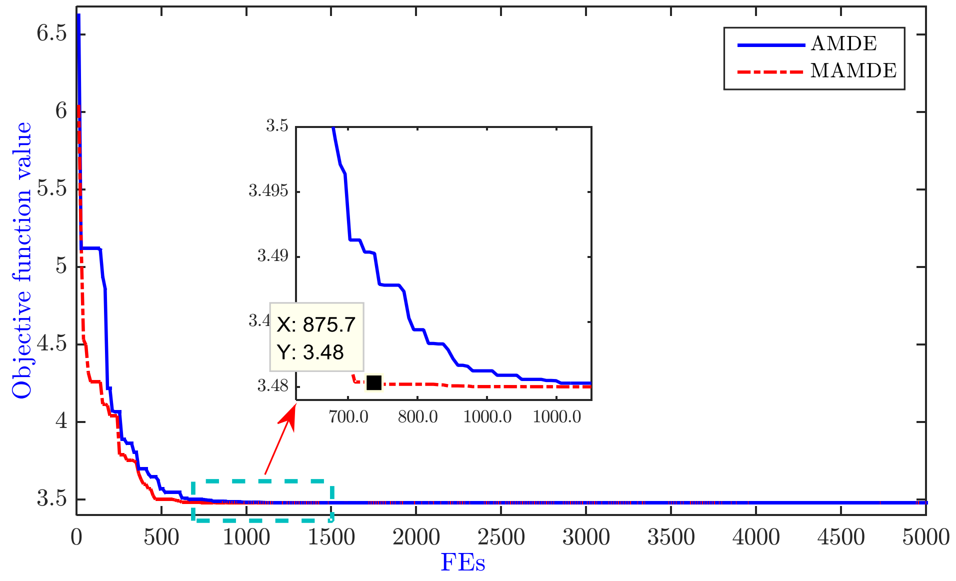


Table 4.11 Statistical results obtained by AMDE and MAMDE for the six mechanical problems

Problem	**Statistics**	**Algorithms**	
		AMDE	**MAMDE**
Multiple clutch disc brake	Best	0.313656610534405	0.313656610534405
	Mean	0.313656610534405	0.313656610534405
	Worst	0.313656610534405	0.313656610534405
	SD	1.68224E-16	**00E+00**
Coupling with a bolted rim	Best	3.48000000000000	3.480000000000000
	Mean	3.48000000000011	3.480000000000000
	Worst	3.48000000000093	3.480000000000000
	SD	1.5736E-13	**1.7943E-15**
Car side impact design	Best	22.842969981655585	22.842970307394101
	Mean	22.842980642247756	22.842978534591708
	Worst	22.843007001312355	22.843002699055756
	SD	7.8986E-06	**7.2561E-06**
Rolling element bearing	Best	-85546.80166904310	-85546.80166904313
	Mean	-85546.80166904128	-85546.80166904129
	Worst	-85546.80166903381	-85546.80166903068
	SD	**2.3827E-09**	2.9055E-09
Step-cone pulley	Best	16.634504852881292	16.634504852868368
	Mean	16.634504860410523	16.634504867131174
	Worst	16.634504929143059	16.634504934566365
	SD	**1.2512E-08**	1.5129E-08
Hydrodynamic thrust bearing	Best	1625.442764500692	1625.442764498245
	Mean	1625.447413771280	1625.442764501321
	Worst	1625.674565029282	1625.442764642627
	SD	0.0327	**2.0411E-08**

Table 4.12 Design variables and constraint values for the mechanical problems using AMDE

	Hydrodynamic thrust bearing	Multiple clutch disc	Car side impact design	Rolling element bearing	Step-cone pulley	Coupling with a bolted rim
			Variables			
x_1	5.955780495321	70.000000000	0.500001229352	125.7196116504	40.0000000183731	2.0000000000
x_2	5.389013045776	90.000000000	1.116182309374	21.42524271844	54.7643008327953	8.0000000000
x_3	0.000005358697	1.0000000000	0.500000235819	11.00000000000	73.0131771121696	59.500000000
x_4	2.269655963392	984.00000000	1.302504626595	0.515000000000	88.4284194672621	40.000000000
x_5	–	3.0000000000	0.500002159032	0.515000000002	85.9862429605478	–
x_6	–	–	1.499998587657	0.444909575881	–	–
x_7	–	–	0.500004105555	0.650805371708	–	–
x_8	–	–	0.345000000000	0.300000000001	–	–
x_9	–	–	0.345000000000	0.063352822105	–	–
x_{10}	–	–	-19.59446532751	0.602943484234	–	–
x_{11}	–	–	0.010051347284	–	–	–
			Constraints			
g_1	−2.4500E-09	0	−0.65499023	1.7599E-04	−1.7393E-10	−0.99998971
g_2	−4.0000E-11	24.00000	−0.07425212	11.7068151	−3.8225E-10	−1.5260103
g_3	−1.0000E-13	0.9083864	−0.06490452	2.70589058	−8.8111E-10	−1.4000E-13
g_4	−3.2436E-04	9.9967854	−319.726880	3.33693819	−0.98686410	−91.9999999
g_5	−0.5667674	9.9649116	−3.71825941	0.71961165	−0.99736014	−40.4999999
g_6	−9.9636E-04	2.3799137	−6.12688598	15.1185938	−1.01015389	−1.0000E-13
g_7	−1.3000E-10	51.095624	−1.4501E-05	8.2929E-12	−1.02059166	−9.50000000
g_8	–	12.620086	−3.1824E-08	3.3306E-16	−698.577271	−59.9999999
g_9	–	–	−0.96593477	2.8100E-12	−475.827141	0
g_{10}	–	–	−0.29673199	1.2988E-14	−209.036936	−17.173000
g_{11}	–	–	–	–	−5.9914E-07	−0.8270000
			Objective functions			
f	1625.442764498	0.3136566105	22.843002277267	−85546.80166	16.63450493407	3.4800000000

Table 4.13 Values of objective functions, variables and constraints for the mechanical problems MADME

	Hydrodynamic thrust bearing	Multiple clutch disc	Car side impact	Rolling element bearing	Step-cone pulley	Coupling with a bolted rim
			Variables			
x_1	5.95578201508552	70.000000000000	0.5000000426403	125.71961165047	40.00000015805	2.00000000000
x_2	5.38901401241576	90.000000000000	1.1164114526723	21.425242718446	54.76430214645	8.00000000000
x_3	0.00000535870405	1.0000000000000	0.5000009679052	11.000000000000	73.01317955476	59.5000000000
x_4	2.26966338267108	984.00000000000	1.3021228285373	0.5150000000000	88.42841861527	40.0000000000
x_5	–	3.0000000000000	0.5000029744484	0.5150000000001	85.98624454239	–
x_6	–	–	1.4999999463920	0.4327902628163	–	–
x_7	–	–	0.5000002332294	0.6831188374053	–	–
x_8	–	–	0.3450000000000	0.3000000000000	–	–
x_9	–	–	0.1920000000000	0.0523374643886	–	–
x_{10}	–	–	-19.55346634580	0.6406383872718	–	–
x_{11}	–	–	-0.003193000080	–	–	–
			Constraints			
g_1	−0.03376536	0	−0.617476108	1.7599E-04	-4.2006E-09	−0.99998971
g_2	−1.0276E-04	24.00000	−0.092703844	12.5551678	−6.2712E-09	−1.52601030
g_3	−1.0817E-04	0.908386	−0.068788927	4.96783318	−3.4878E-09	−3.0000E-14
g_4	−3.2436E-04	9.996785	−319.7141824	2.20609110	−0.98686410	−91.9999999
g_5	−0.56676800	9.964911	−4.278015795	0.71961165	−0.99736014	−40.4999999
g_6	−9.9636E-04	2.379913	−7.282676164	12.3647544	−1.01015389	−1.0000E-14
g_7	−4.3039E-03	51.09562	−1.00285E-05	2.3000E-12	−1.02059166	−9.50000000
g_8	–	12.62008	−4.2452E-07	1.0000E-14	−698.577299	−59.9999999
g_9	–	–	−0.96522844	1.0000E-13	−475.827185	0
g_{10}	–	–	−0.16665422	3.0000E-14	−209.036975	−17.173000
g_{11}	–	–	–	–	−5.4836E-06	−0.8270000
			Objective functions			
f	1625.442764498	0.3136566105344	22.84298982791	−85546.8016690	16.634505631286	3.48000000000

Fig. 4.22 Convergence graph of AMDE and MAMDE for car side impact design problem

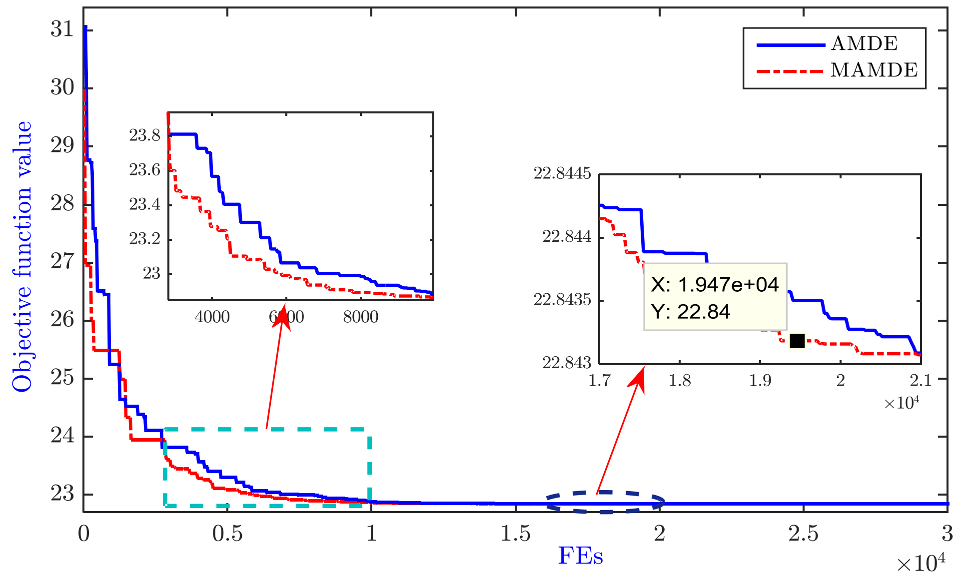


Fig. 4.23 Convergence graph of AMDE and MAMDE for rolling element bearing design problem

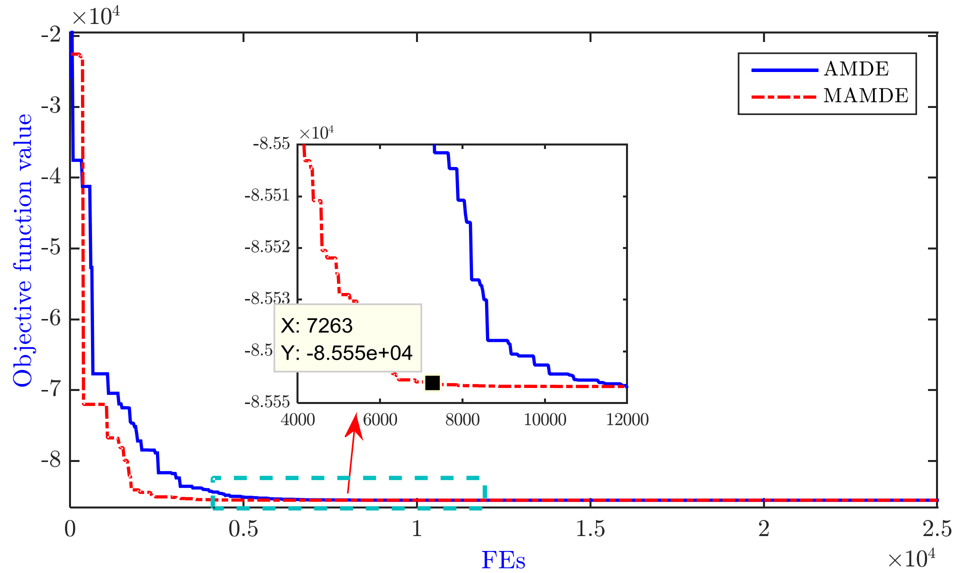


Fig. 4.24 Convergence graph of AMDE and MAMDE for step-cone pulley design problem

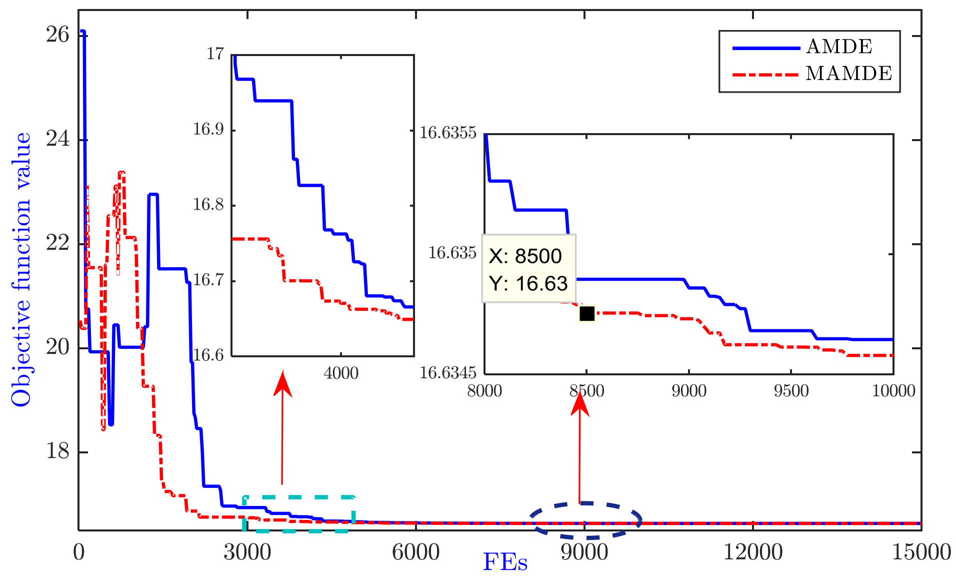
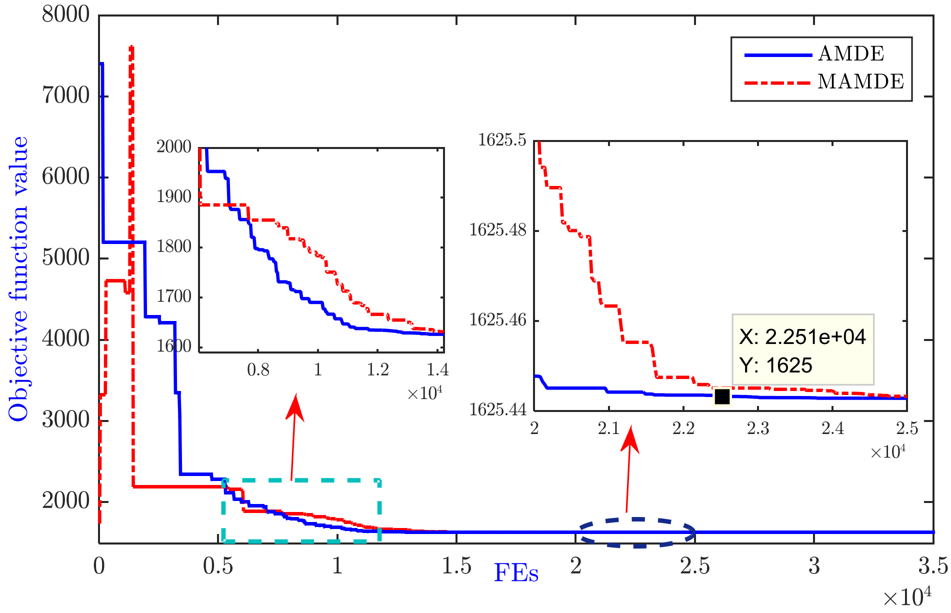


Fig. 4.25 Convergence graph of AMDE and MAMDE for hydrostatic thrust bearing design problem



4.4.3.2. Comparison with other algorithms

The multiple disc clutch brake problem has been solved using WCA (Eskandar et al., 2012), TLBO and ABC (Rao et al., 2011). The statistical optimization results for the five methods are presented in Table 4.14. According to Table 4.14, we can see that TLBO and ABC are not able to get the best reported solution in all runs. In terms of the standard deviation, MAMDE is the best one with value of zero. AMDE and WCA provided the same standard deviation. However, WCA is most efficient in solving this problem with 500 FEs.

Table 4.14 Comparison with other state-of-the-art algorithms on multiple disc clutch brake problem

Algorithm	Best	Mean	Worst	SD	FEs
MAMDE	0.313656610534405	0.313656610534405	0.313656610534405	**00E+00**	1,500
AMDE	0.313656610534405	0.313656610534405	0.313656610534405	1.68224E-16	1,500
WCA	0.313656	0.313656	0.313656	1.69E-16	**5,00**
TLBO	0.313657	0.3271662	0.392071	0.67	NA
ABC	0.313657	0.324751	0.352864	0.54	NA

The problem of the coupling with a bolted rim was solved by Giraud-Moreau and Lafon (2002) using the genetic algorithm and evolution strategy. From the statistical results shown in Table 4.15, it can be seen that the proposed approaches find a new solution for this problem (3.48000). The best value of the objective function is improved by 10 % compared with GA and ES. Moreover, MAMDE provide the smallest standard deviation with value of 1.7943E-15 followed by AMDE with value of 1.5736E-13.

Table 4.15 Comparison with other state-of-the-art algorithms on coupling with a bolted rim problem

Algorithm	Best	Mean	Worst	SD	FEs
MAMDE	**3.48000000000000**	**3.48000000000000**	**3.48000000000000**	**1.7943E-15**	7,000
AMDE	3.48000000000000	3.48000000000011	3.48000000000093	1.5736E-13	7,000
GA	3.8797	NA	NA	0.0183	5,320
ES	3.8797	NA	NA	0.0675	**2,120**

The car side impact design is considered as a real case of mechanical optimization problem with mixed discrete and continuous design variables. In this case AMDE and MAMDE are compared with εDE-LS, PSO, Firefly Algorithm (Gandomi et al., 2011), and Cuckoo Search (CS) algorithm (Gandomi et al., 2013). The statistical results are given in Table 4.16, it can be seen that the best solution for the problem is obtained by CS. In terms of the robustness the εDE-LS is the best one followed by our proposed approaches. However, the best solution find by AMDE is better than of the εDE-LS. Our algorithms required more FEs for the problem of car impact side design.

The rolling element bearing was optimized previously using Rank-iMDDE, TLBO, ABC, Mine Blast Algorithm (MBA) (Sadollah et al., 2013), WCA and GA (Gupta et al., 2007). The statistical optimization results for the reported algorithms are summarized in Table 4.17. It's clearly observed that AMDE and MAMDE obtain the best results compared with other optimizers in terms of best (-85546.801669), mean and worst solutions and standard deviation. The AMDE can achieve the best results in terms of mean and worst compared with MAMDE.

Table 4.16 Comparison with other state-of-the-art algorithms on car side impact problem

Algorithm	Best	Mean	Worst	SD	FEs
MAMDE	22.842970307394101	22.842978534591708	22.843002699055756	7.2561E-06	30,000
AMDE	**22.842969981655585**	22.842980642247756	22.843007001312355	7.8986E-06	30,000
εDE-LS	22.84297	22.84297	22.84297	**5.60176E−07**	20,000
CS	22.84294	22.85858	23.25998	0.07612	NA
FA	22.84298	22.89376	24.06623	0.16667	NA
PSO	22.84474	22.89429	23.21354	0.15017	NA

Table 4.17 Comparison with other state-of-the-art algorithms on rolling element bearing problem

Algorithm	Best	Mean	Worst	SD	FEs
MAMDE	**-85546.80166904313**	**-85546.80166904129**	**-85546.80166903068**	**2.9055E-09**	25,000
AMDE	**-85546.80166904310**	**-85546.80166904128**	-85546.80166903381	**2.3827E-09**	25,000
Rank-iMDDE	−81859.732421	−81859.010377	−81838.757577	NA	10,000
MBA	−85535.9611	−85321.4030	−84440.1948	211.52	15,100
WCA	−85538.48	−83847.16	−83942.71	488.30	**3,950**
TLBO	−81859.740000	−81438.987000	−80807.855100	NA	10,000
ABC	−81859.741600	−81496.000000	−78897.810000	NA	10,000
GA	−81843.3	NA	NA	NA	225,000

For the step-cone pulley problem the AMDE and the MAMDE are compared with TLBO and ABC, the statistical results for the four metaheuristics are tabulated in Table 4.18. According to the results the best, mean and worst solutions found by AMDE and MAMDE are better than those obtained by TLBO and ABC. Further, our algorithms can also obtain the lower standard deviation (1.5129E-08). Accordingly, it can be said that AMDE and MAMDE are very effective to solve this problem.

The hydrostatic thrust bearing problem was previously solved using Rank-iMDDE, TLBO and ABC. The obtained statistical results for AMDE, MAMDE and the three compared algorithms are given in Table 4.19. From the Table, it can be observed that the best solution is obtained by ABC, followed by MAMDE and AMDE. However, MAMDE is the most robust among the compared algorithms with standard deviation value of 2.0411E-08. The mean and the worst solutions obtained by MAMDE and AMDE are better than of Rank-iMDDE, TLBO and ABC.

Table 4.18 Comparison with other state-of-the-art algorithms on step-cone pulley problem

Algorithm	Best	Mean	Worst	SD	FEs
MAMDE	**16.634504852868368**	**16.634504867131174**	**16.634504934566365**	**1.5129E-08**	15,000
AMDE	**16.634504852881292**	**16.634504860410523**	**16.634504929143059**	**1.2512E-08**	15,000
TLBO	16.634510	24.011358	74.022951	NA	15,000
ABC	16.634655	36.099500	145.470500	NA	15,000

Table 4.19 Comparison with other state-of-the-art algorithms on hydrostatic thrust bearing problem

Algorithm	Best	Mean	Worst	SD	FEs
MAMDE	1625.442764498245	1625.442764501321	1625.442764642627	**2.0411E-08**	35,000
AMDE	1625.442764500692	1625.447413771280	1625.674565029282	0.0327	35,000
Rank-iMDDE	1625.460142	1724.727935	1894.734127	NA	**25,000**
TLBO	1625.443000	1797.707980	2096.801270	NA	**25,000**
ABC	**1625.442760**	1861.554000	1861.554000	NA	**25,000**

4.5. Conclusion

In this chapter, the effectiveness of the AMDE and MAMDE are illustrated thorough twelve structural engineering design optimization problems including six engineering benchmarks and six mechanical problems. For the optimization procedure the two developed methods require reasonable FEs number to achieve the feasible global optimal solution, no more than 150,000 in case the stepped cantilever beam (the first set of problems) and no more than 35,000 in case the hydrodynamic thrust bearing (for the second type of test problem). Further, the used index technique is very effective to handle the integer and the discrete variables.

The comparative results between the two developed algorithms showed that the MAMDE is more robust than the AMDE version in solving the twelve engineering problems. In addition, the convergence speed to the global solution of the MAMDE is faster than the AMDE. This indicates and confirms that the superiority of feasible points mechanism used in the selection step is better than the classical one which based only on the fitness value to compare between the individuals.

For the first six engineering benchmarks, the optimal results given by our algorithms are compared with eighteen recent published meta-heuristics. The comparison results indicate that the AMDE and MAMDE are competitive with, and in some cases superior to, other existing algorithms in terms of the quality, efficiency, and robustness of the final solution. For the mechanical design problems our developed variants are compared with eight methods. The simulation results illustrate that the proposed algorithms can achieve the optimal global solution. In terms of the standard deviation our algorithms provides the smallest values for all problems. In addition, they produced improved results for the coupling with a bolted rim, the rolling element bearing design and the step-cone pulley design problem. Thus, we can conclude that the MAMDE and AMDE are important alternatives to solve real-world optimization problems.

CHAPTER FIVE:

Cylindrical gears design optimization

CHAPTER FIVE

CYLINDRICAL GEARS DESIGN OPTIMIZATION

5.1. Introduction

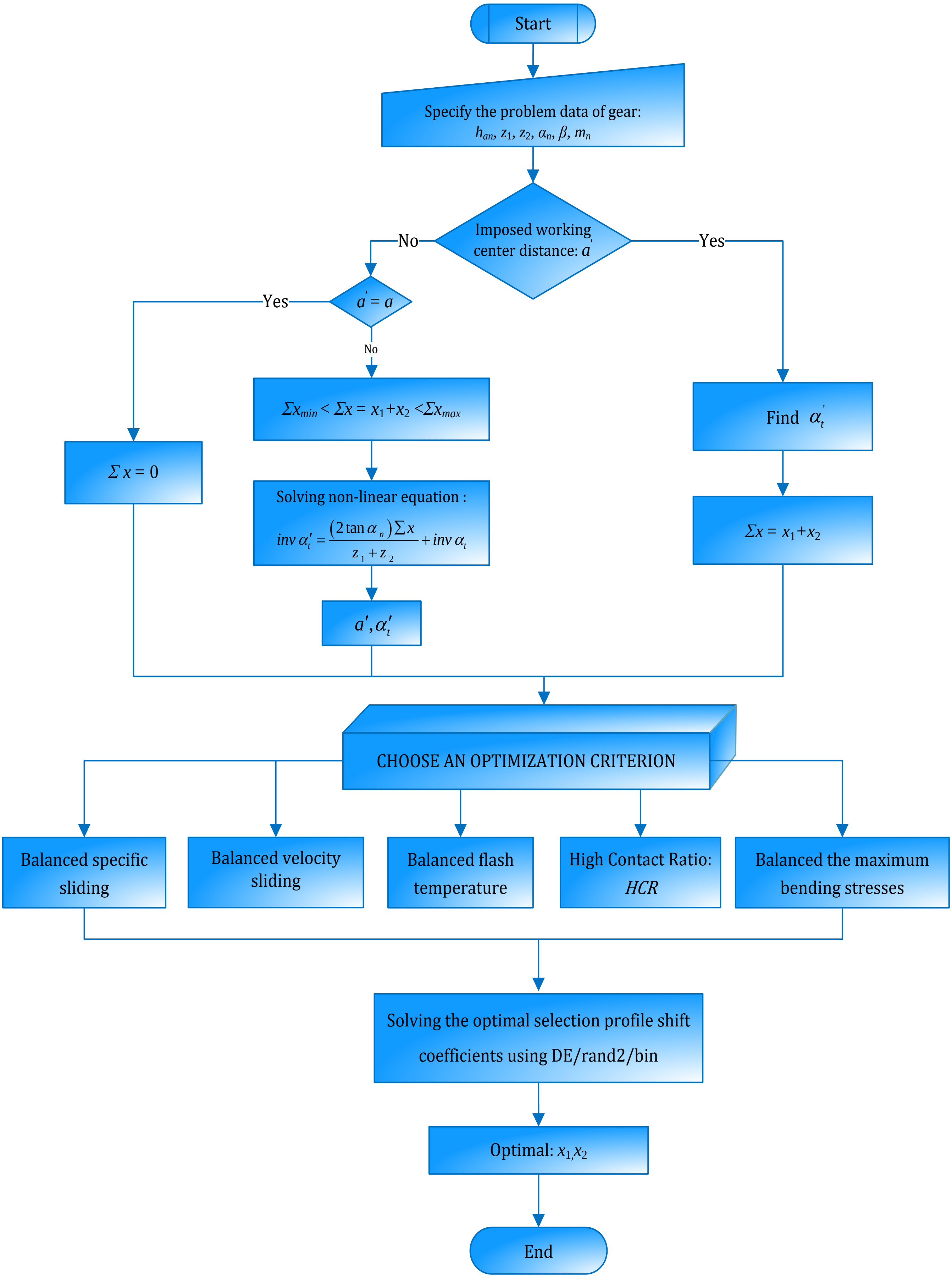
In the present chapter we applied the proposed methods to solve the problem of the optimal geometry of the cylindrical gears. In the first application, the optimal selection values of profile shift coefficients for cylindrical involute spur and helical gears is performed using the DE/rand2/bin version. In this part the optimization procedure is developed specifically for exact balancing specific sliding coefficients at extremes of contact path and account for gear design constraints. In the second step the AMDE and the MAMDE algorithms are applied to optimize the profile tooth of a real spur gear in a large transport machine, where the objectives are the wear resistance and maximum bending stresses. Finally, the developed algorithms are applied in order to solve a single stage cylindrical speed reducer. The mathematical formulation of this problem takes into account all the standards recommendations specified for complete gear design.

5.2. Optimal selection of profile shift coefficients

As already mentioned in the second chapter (section: importance of profile shift of the teeth), the gear correction is carried out in order to achieve of non standard distances between shafts, to improve the loading capacity and to avoid narrow top lands of teeth.

In this section, the optimal selection values of profile shift coefficients for cylindrical involute spur and helical gears is described, using a DE/rand2/bin version for different cases of gear correction. For more explanation the flow chart presented in the Fig. 5.1 summarized the search algorithm in order to find the optimal coefficients for different optimization criterion. The design variables to be optimized in this case are profile shift coefficients x_1 and x_2. The optimization procedure is investigated for multiple objectives among them the specific sliding, velocity sliding, flash temperature, high contact ratio, and bending stresses. In this formulation we considered that pinion and wheel number of teeth, module and other parameters as input data. Furthermore, the minimal transverse contact ratio value, thickness of the tooth tip diameter and tooth interferences are considered as constraints functions. After the formulation of the optimization procedure and selection of the fitness term, the DE algorithm is used to solve the problem of the optimal profile shift coefficient. We should mention that among the existing objectives, in the following we chose to show only the results of balancing the specific sliding.

Fig. 5.1 Algorithm search for optimal selection of profile shift coefficients



5.2.1. Design objective function

The optimization problem formulation considered two continuous variables x_1 and x_2, and takes into account for all design constraints that influence the geometry of a gear pair. The specific sliding is the main geometrical parameter that affects scuffing and the teeth gear wear. In order to maximize the service life and wear resistance of the gear pair, the maximum specific sliding coefficients must be equal at extremes of contact path (the points *A* and *E* see Fig. 2.6). So, the objective function can be expressed as:

$$f(x) = |\gamma_{2\max} - \gamma_{1\max}| \tag{5.1}$$

The maximal specific pinion sliding coefficient γ_{1max} is given by:

$$\gamma_{1\max} = \frac{\tan\alpha_{at1} - \tan\alpha_t'}{(1+u)\tan\alpha_t' - \tan\alpha_{at1}}(u+1) \tag{5.2}$$

-The maximal specific wheel sliding coefficient γ_{2max} is given by:

$$\gamma_{2\max} = \frac{\tan\alpha_{at2} - \tan\alpha_t'}{\left(1+\frac{1}{u}\right)\tan\alpha_t' - \tan\alpha_{at2}}\left(\frac{u+1}{u}\right) \tag{5.3}$$

where $x = \{x_1, x_2\}$ is the vector of design variables, u is the transmission ratio, α_t' is pressure angle at the pitch cylinder, and α_{at1}, α_{at2} are respectively the tip transverse pressure angles of pinion and wheel. According to Henriot (2007), balancing specific sliding coefficients leads also to balancing Almen factor. The latter represents a product of the Hertzian contact stress and the sliding velocity ($\sigma_H \times v_g$).

5.2.2. Design constraints

The above objective function is subjected to the following constraints:

- For obtaining a stable and silent transmission, the minimal transverse contact ratio value ε_α must be greater than unity. However, in practice to reduce vibration, contact ratio must be greater than 1.3 for spur gear and 1.1 for helical gear (Pedrero and Artés, 1996).

$$g_1(x) = \begin{cases} 1.3 - \varepsilon_\alpha \le 0 \; for \;\; spur \;\; gear \\ 1.1 - \varepsilon_\alpha \le 0 \; for \;\; helical \;\; gear \end{cases} \tag{5.4}$$

- To avoid narrow top lands of teeth (Nicolet, 2006), the thickness of the tooth tip diameter should be greater than or equal to 0.4 m_t.

- For pinion:

$$g_2(x) = 0.4m_t - s_{at1} \le 0 \tag{5.5}$$

- For wheel:

$$g_3(x) = 0.4m_t - s_{at2} \le 0 \tag{5.6}$$

where m_t is the transverse module, s_{at1} and s_{at2} are the transversal arc thickness of the tooth at the tip diameter for the pinion and the wheel respectively.

- To avoid operating interference on the two mating gears, the distances O_2N_1 and O_1N_2 should be respectively greater than the tip diameters of the mating gears.

- Pinion interference:

$$g_4(x) = \frac{m_t(z_2 + 2 + 2x_2)}{2} - \sqrt{{r_{b2}}^2 + \left(a' \sin \alpha_t'\right)^2} < 0 \tag{5.7}$$

- Wheel interference:

$$g_5(x) = \frac{m_t(z_1 + 2 + 2x_1)}{2} - \sqrt{{r_{b1}}^2 + \left(a' \sin \alpha_t'\right)^2} < 0 \tag{5.8}$$

where, a' is the working center distance, r_{b1} and r_{b2} are respectively base radii of pinion and wheel.

- To avoid undercutting in the mating gears, the following relations must be checked

$$g_6(x) = \frac{z_{\min} - z_1}{z_{\min}} h_{an}^* - x_1 \le 0 \tag{5.9}$$

$$g_7(x) = \frac{z_{\min} - z_2}{z_{\min}} h_{an}^* - x_2 \le 0 \tag{5.10}$$

where $z_{\min}$ is the limit for the number of teeth to avoid undercut, and h_{an}^* is the addendum coefficient. $z_{\min}$ is defined in Eq. 2.9 and Eq.2.10.

- Maximum calculated normal and bending stresses should be less than the permissible values for the particular material. The stresses were calculated in this study according to "Method B" of the standard ISO 6336-3:2006.

$$\sigma_{Hi\max} - \sigma_{Hi\lim} \le 0 \tag{5.11}$$

$$\sigma_{Fi\max} - \sigma_{Fi\lim} \le 0 \tag{5.12}$$

- In order to have a meshing without clearance, the sum of profile shift coefficients must satisfy the following equation:

$$x_1 + x_2 = \frac{z_1 + z_2}{2 \tan \alpha_n} \left(inv\, \alpha_t' - inv\, \alpha_t \right) \tag{5.13}$$

5.2.3. Experimental study

In this section the DE/rand2/bin algorithm is used to find the optimal profile shift coefficients for an arbitrary external involute spur and helical gears. The pinion and wheel number of teeth and the sum of profile shifts were used as input data. The mathematical model is established based on the principles of equalized specific sliding at extremes of contact path in order to maximize the service life of the gear pair (Abderazek et al, 2015).

In the first step, the performance of the proposed method is demonstrated by comparing the optimal results obtained by DE with those of Ulaga et al. (2001) using Adaptive Grid-refinement algorithm (AGr) and PD 6457:1970 standard. These methods are classified as heuristic deterministic methods and PD 6457:1970 standard using computation formulae (see Eq. 2.19 in the second chapter). The control parameters of the DE are: population size $NP = 20$, generations $G_{max} = 50$, F and Cr are randomly generated within [0.5, 1.0] and [0.95, 1.0], respectively.

5.2.3.1. Comparison results between DE and AGr

The allocation of the profile shift coefficients is performed for a gear pair analyzed previously by Ulaga et al. (2001), using AGr algorithm. The main characteristics of the analyzed involute spur gear pair are: number of teeth $z_1 = 16$; $z_2 = 24$; face width $b = 14\, mm$; module $m_n = 4.5\, mm$ and pressure angle $\alpha_n = 20°$.

The objective function evaluation, using DE, over 50 generations is shown in Fig. 5.2. The optimal feasible solution found by DE is $f = 0$ for x_1 = 0.3035 and x_2 = 0.0497. The comparisons results are presented in Table 5.1. It can be clearly seen that the profile shift cofficients values obtained by DE lead to a perfect balancing of the specific sliding coefficients ($\gamma_{1max} = \gamma_{2max}$ = 2.5311). Whereas by using AGr, the specific sliding coefficients are not equal (γ_{1max} = 2.5449 and γ_{2max} = 2.5072), which may not make a regulated resistance to wear. It can be said that the DE/rand/2/bin version is more efficient than the AGr algorithm in solving this problem. In addition, from the Fig. 5.2, it is shown that after only 10 iterations the DE algorithm converges to the near-optimum solution.

By using the optimal design parameters obtained by DE for the analyzed gear example, the evolution of specific sliding distribution along the contact path is plotted in Fig.5.3. From the figure it can clearly observed that specific sliding at the tip and the root of the teeth is balanced, such as at the tip ($\gamma_{1max\,(E)} = \gamma_{2max\,(A)}$ = 0.7175) and for the root ($\gamma_{1max\,(E)} = \gamma_{2max\,(A)}$ = 2.5311).

Table 5.1 Optimization results of DE and AGr

Variables	DE/rand/2/bin	AGr
x_1	0.3035	0.3070
x_2	0.0497	0.0460
g_1	-0.3644	-0.3639
g_2	-1.1007	-1.0909
g_3	-1.8991	-1.9034
g_4	-5.9328	-5.9647
g_5	-13.692	-13.659
g_6	-0.2389	-0.2425
g_7	-0.5428	-0.5391
$f_{\min}$	0.0000	0.0377

Fig. 5.2 Convergence of DE/rand2/bin algorithm to the best fitness respect to the generations

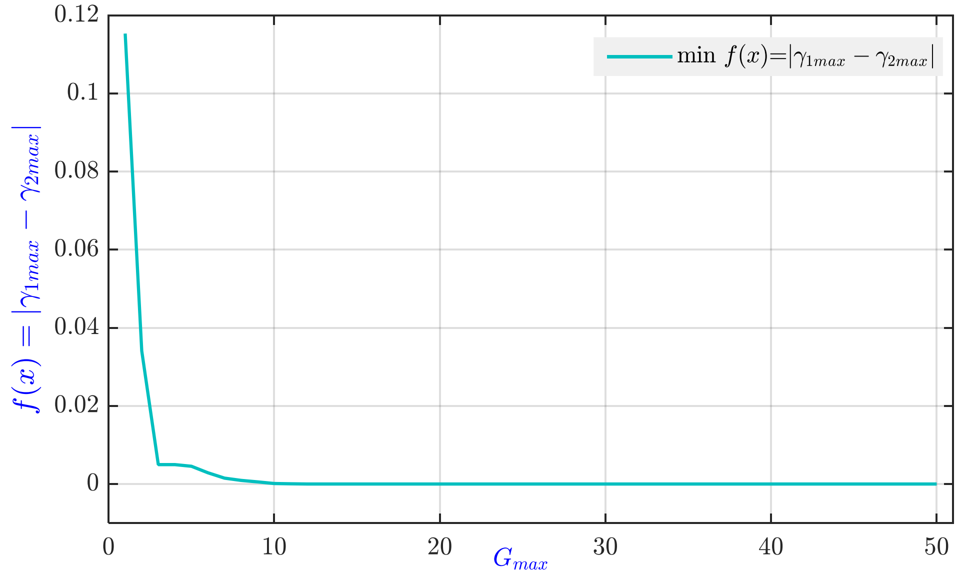
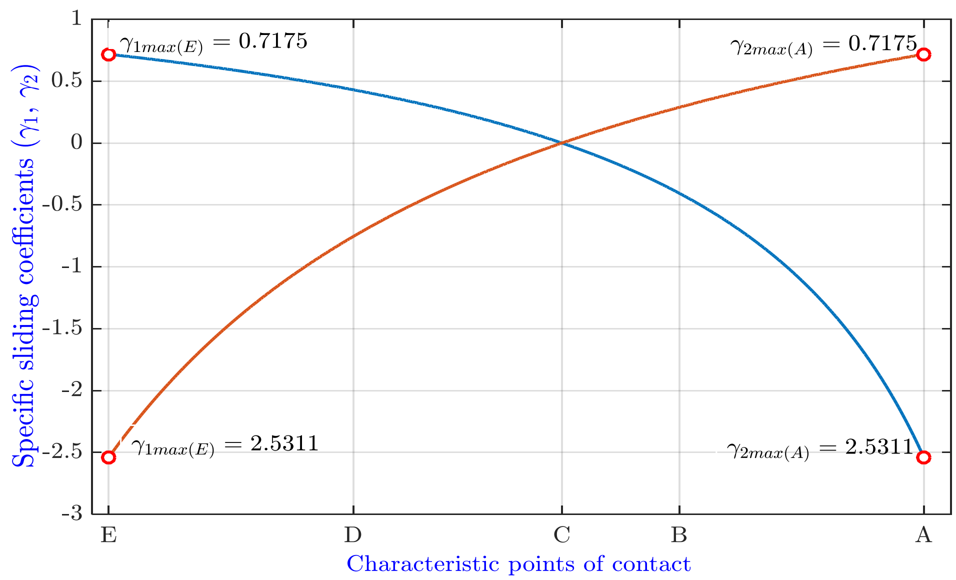


Fig. 5.3 Evolution of specific sliding distribution along the contact path



5.2.3.2. Comparison results between DE and PD 6457 standard

The results obtained by DE algorithm, in order to optimize the profile shift coefficients for two external cylindrical gears one is spur and other is helical, are compared with those provided by PD 6457: 1970 uses a computation formulae. The characteristics of the gears and comparison results between both methods are given respectively in Table 5.2 and Table 5.3. The objective function evaluation, using DE, over the generations for the two examples is shown in Figs. 5.3 and 5.4 respectively.

Acording the results of the Table 5.3, the profile shift cofficients obtained by our method DE make sure that the two gears have the same specific sliding coefficients for both examples ($\gamma_{1max} = \gamma_{2max}$ = 2.3132 for the spur gear and $\gamma_{1max} = \gamma_{2max}$ = 2.712613 for the helical gear), which resulting a balanced resistance to wear. But using PD 6457: 1970 computation formulae, the specific sliding coefficients are not actually equal. In addition, as showen in Figs. 5.4 and 5.5 it can be said that DE converged rapidly and can achieve the near-optimum within only 10 iterations for both cases.

Based on the optimal solution obtained by DE for the two analyzed gears, the evolution of specific sliding distribution along the contact path for the spur and the helical gears is plotted in Fig. 5.6 and Fig. 5.7, respectively. It can be seen that the specific sliding coefficients at extremes of contact path are equal for both cases. For the spur gear ($\gamma_{1max\,(E)} = \gamma_{2max\,(A)}$ = 0.668, and $\gamma_{1max\,(E)} = \gamma_{2max\,(A)}$ = 2.001) and for the helical gear ($\gamma_{1max\,(E)} = \gamma_{2max\,(A)}$ = 0.7306, and $\gamma_{1max\,(E)} = \gamma_{2max\,(A)}$ = 2.713).

Table 5.2 Gears characteristics

z_1	z_2	$\alpha_n(°)$	$\beta(°)$	m_n(mm)	h^*_{an}	$x_1 + x_2$
15	18	20	0	6	1	0.8
14	16	20	16	4	1	0.5

Table 5.3 Comparison results between DE/rand2/bin and PD 64577

	Spur gear		Helical gear	
Variables	**DE/rand2/bin**	**PD 6457**	**DE/rand2/bin**	**PD 6457**
x_1	0.42944	0.44970	0.28828	0.29630
x_2	0.37055	0.35029	0.21171	0.20369
γ_{1max}	**2.00093**	2.06822	**2.712613**	2.75355
γ_{2max}	**2.00093**	1.91807	**2.712613**	2.66329

Fig. 5.4 Convergence of DE algorithm to the best fitness respect to the generations for the spur gear

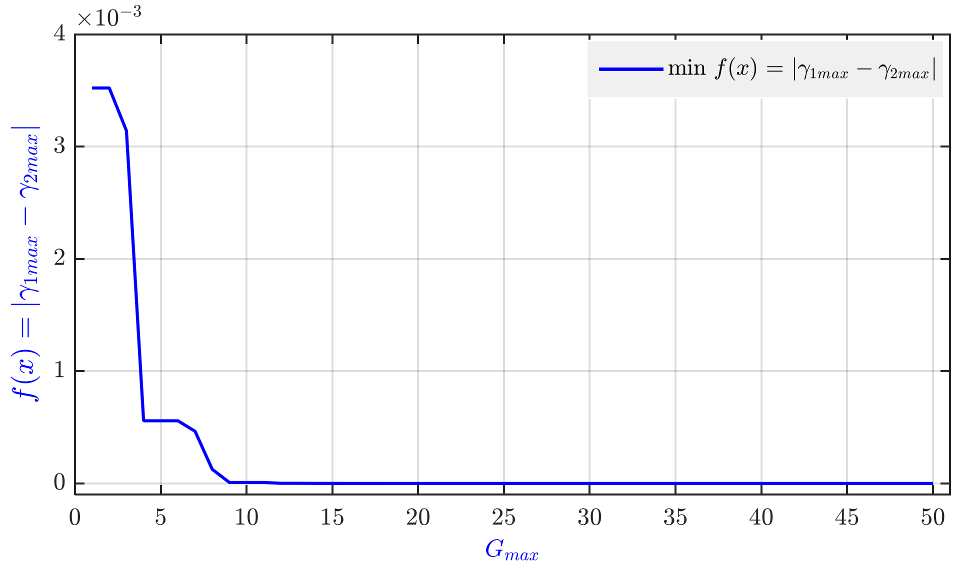


Fig. 5.5 Convergence of DE algorithm to the best fitness respect to the generations for the helical gear

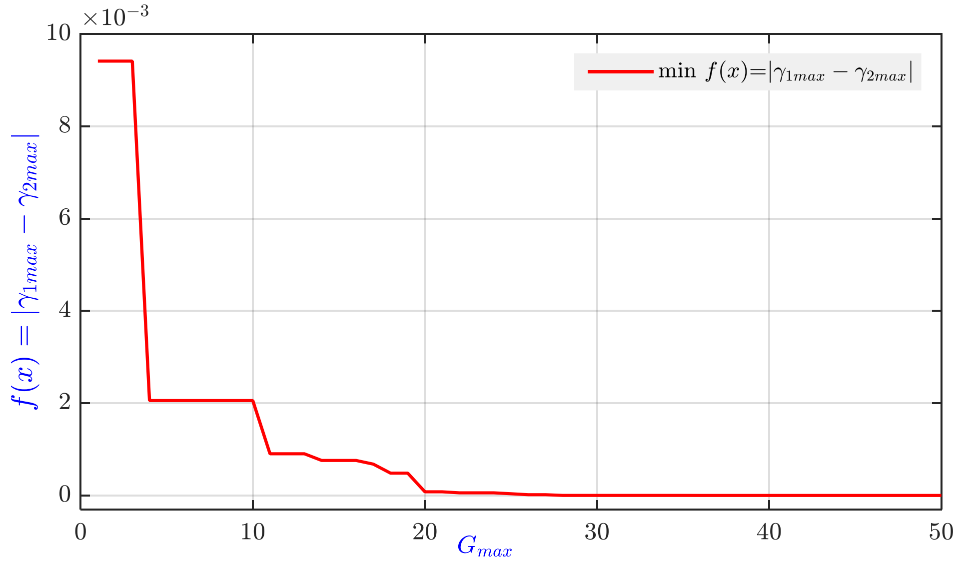


Fig. 5.6 Evolution of specific sliding distribution along the contact path for the spur gear

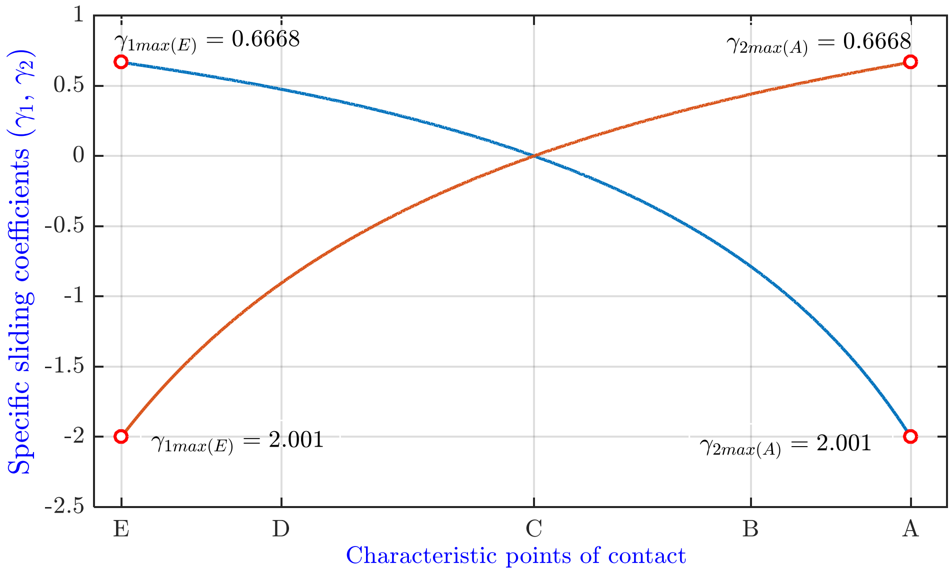
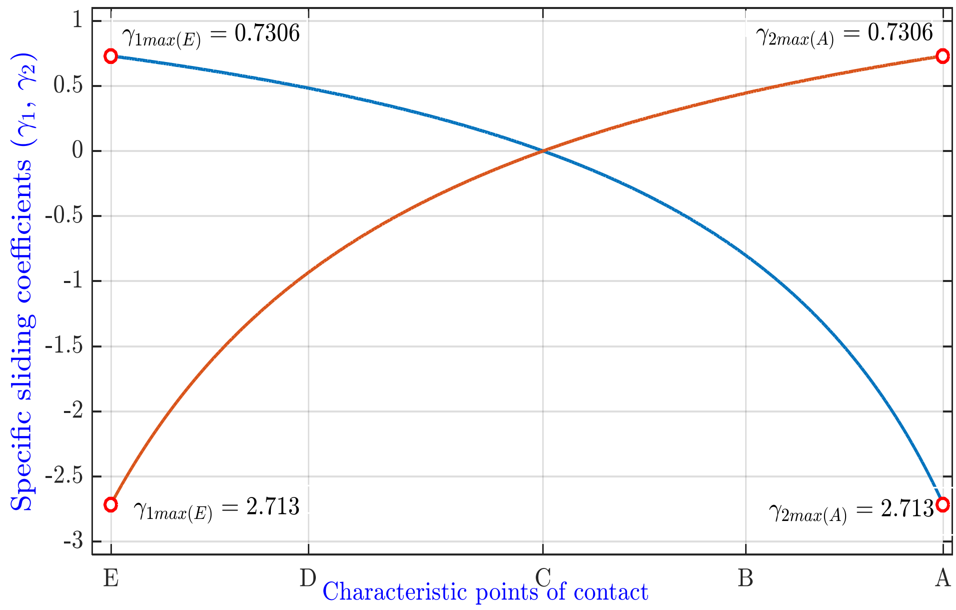


Fig. 5.7 Evolution of specific sliding distribution along the contact path for the helical gear



5.2.4. Distribution of the profile shift coefficients by DE

Figures 5.8 to 5.13 gives an intuitive idea about the accuracy of the proposed method. Pinion profile coefficient have been represented as a function of the velocity ratio that equalizing specific sliding coefficients in both gears and these for several values of pinion tooth number, so they can be easily used as charts for gear manufacturers and designers.

The following data ranges are considered:

- Numbers of pinion teeth:

$z_1 \in \{13; 14; 15; 16; 17; 18; 19; 20\}$

- Gear ratios:

$u \in \{1.4; 1.6; 1.8; 2; 2.24; 2.5; 2.8; 3.15; 3.55; 4; 4.5; 5; 5.6; 6.3; 7.1; 10\}$

- Sums of profile shift coefficients:

$\sum x \in \{0; 0.5; 0.8; 1\}$

- Pressure angle:

$\alpha = 20°$

- Helical angle values:

$\beta \in \{0°; 16°\}$

Fig. 5.8 Profile coefficient x_1 for $\sum x = 0.5$, $\alpha = 20°$ and $\beta = 0°$

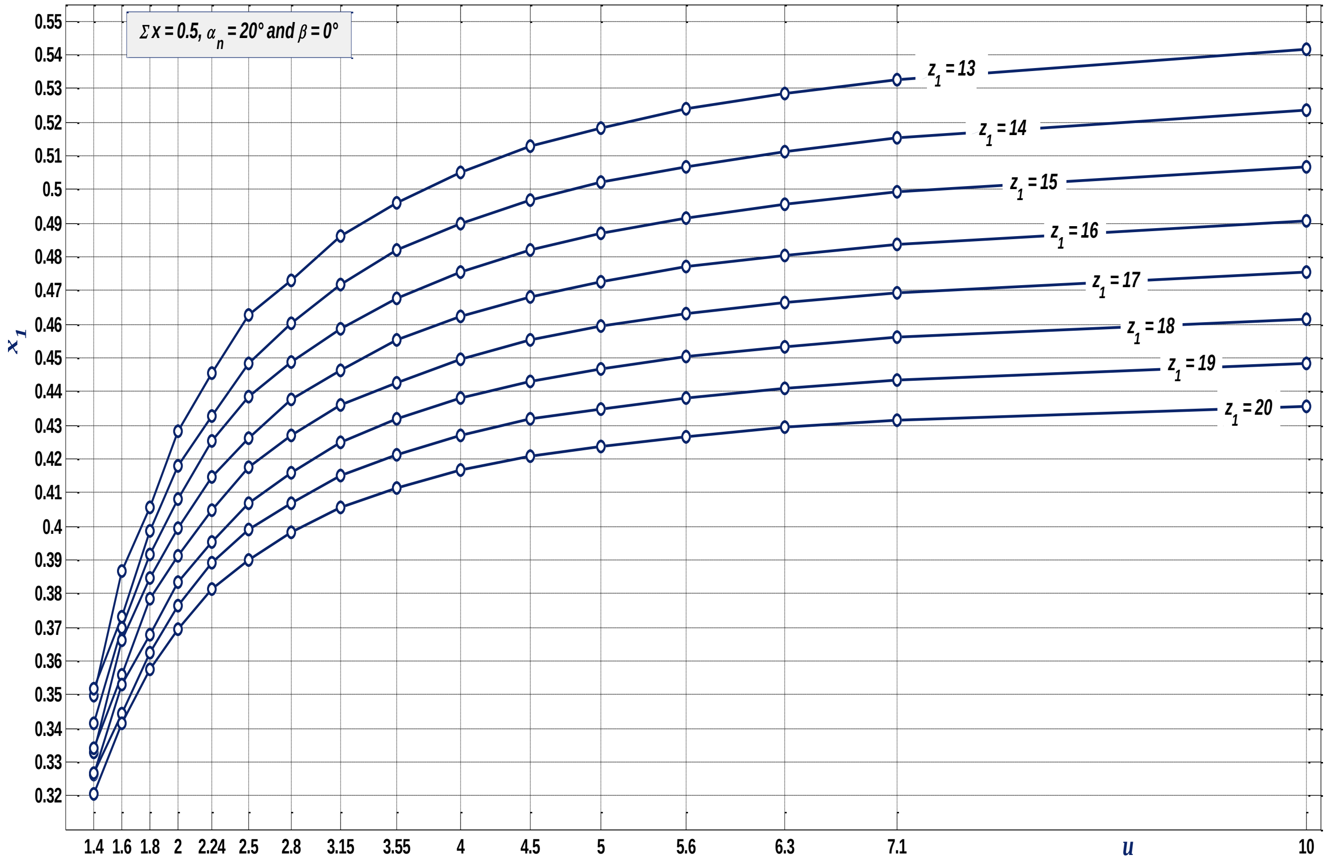


Fig. 5.9 Profile coefficient x_1 for $\sum x = 0.8$, $\alpha = 20°$ and $\beta = 0°$

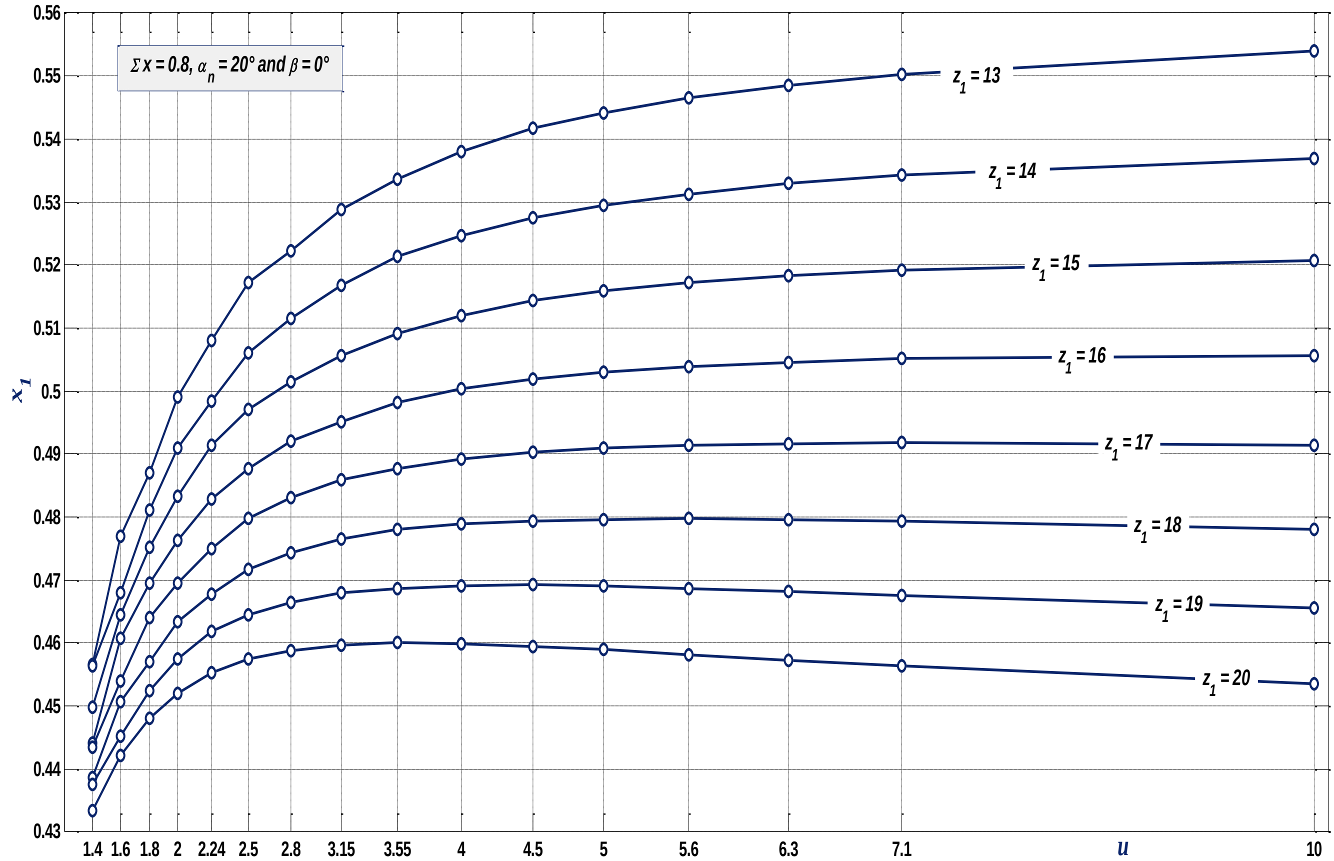


Fig. 5.10 Profile coefficient x_1 for $\sum x = 1$, $\alpha = 20°$ and $\beta = 0°$

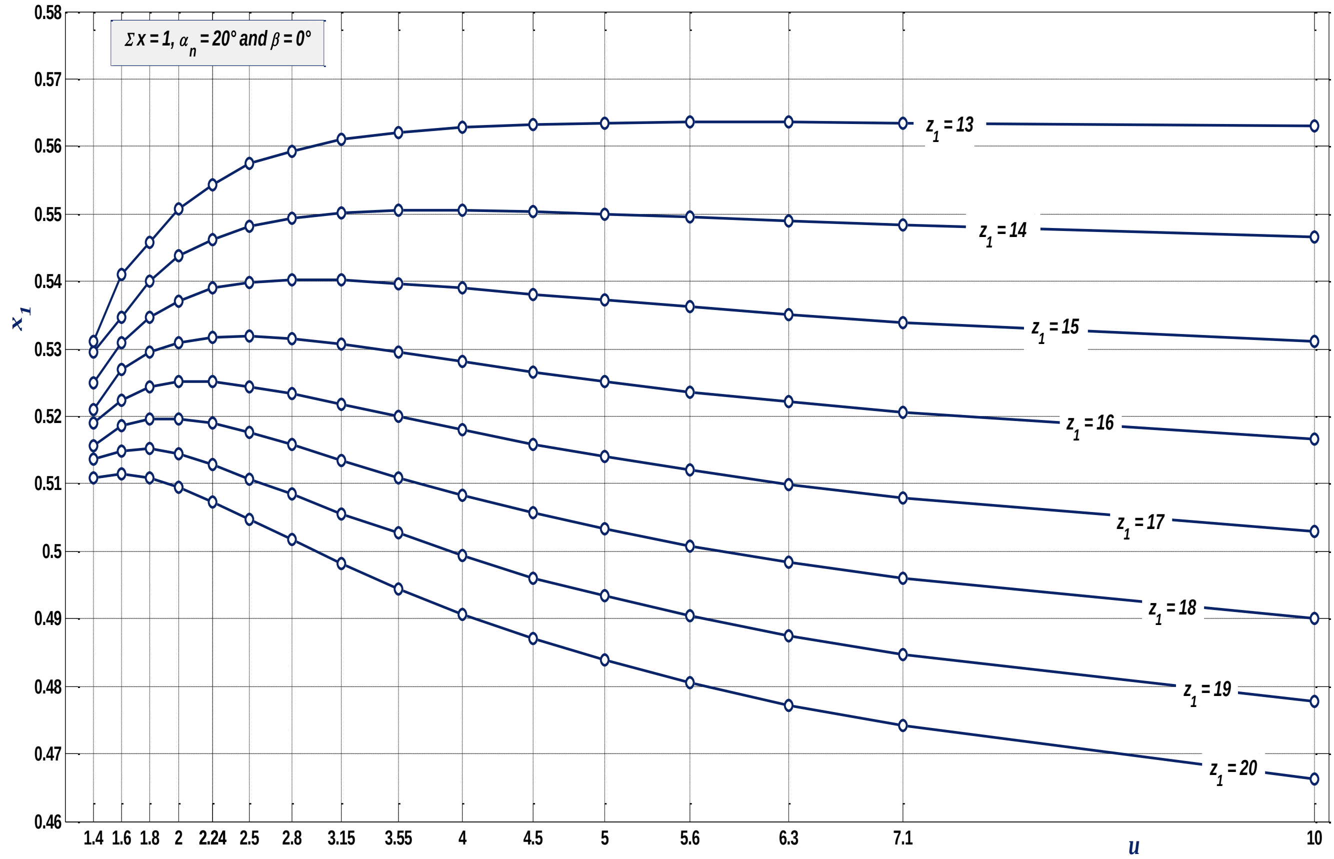


Fig. 5.11 Profile coefficient x_1 for $\sum x = 0.5$, $\alpha = 20°$ and $\beta = 16°$

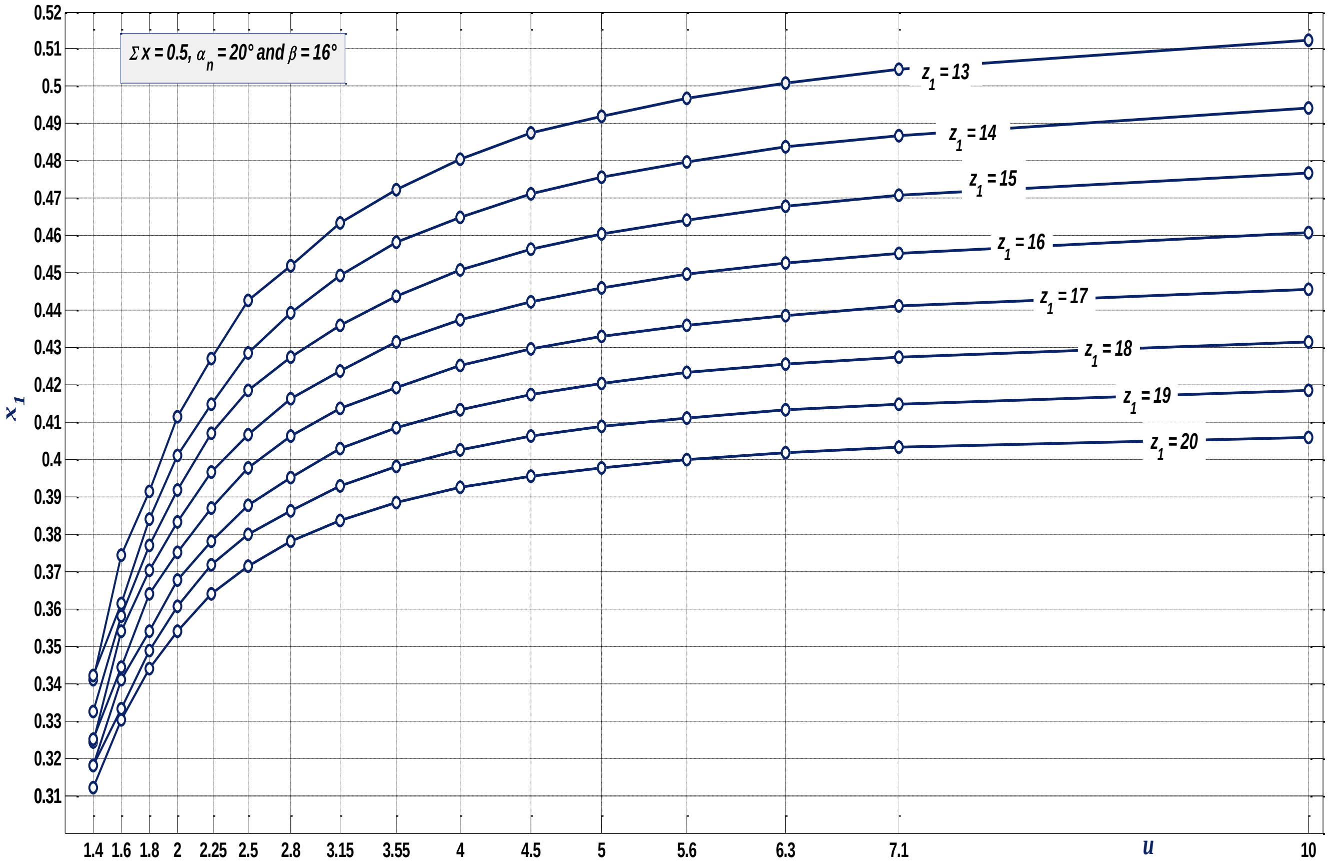


Fig. 5.12 Profile coefficient x_1 for $\sum x = 0.8$, $\alpha = 20°$ and $\beta = 16°$

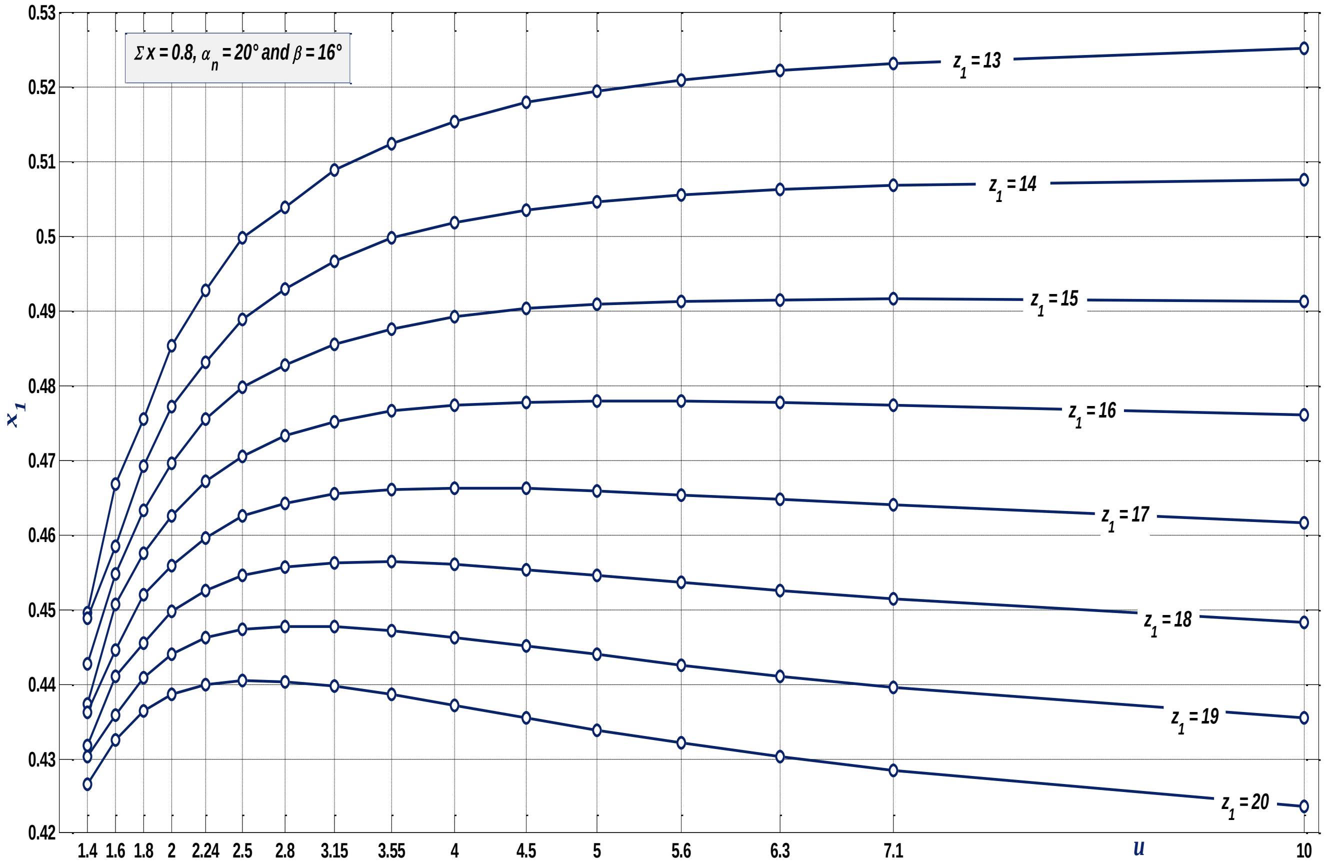
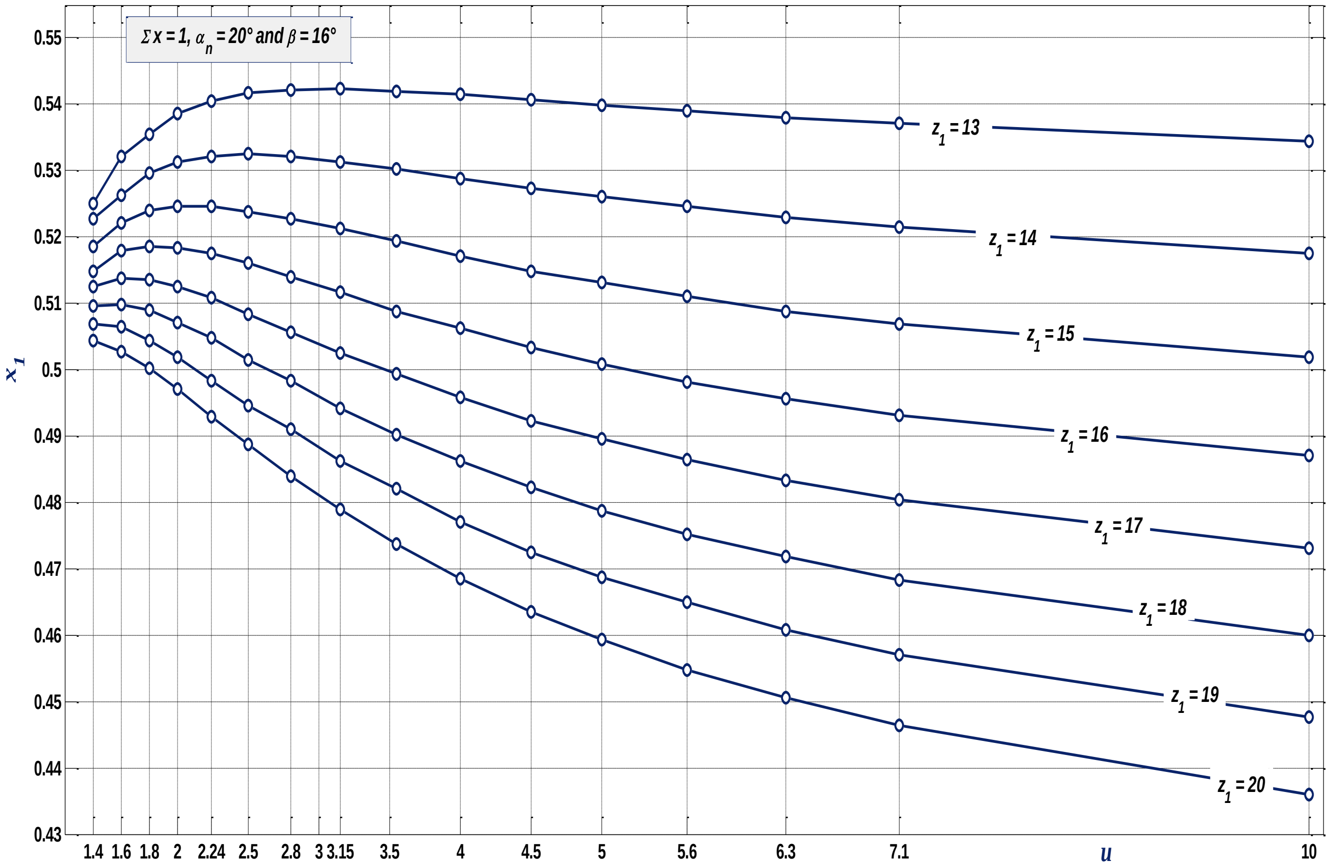


Fig. 5.13 Profile coefficient x_1 for $\sum x = 1$, $\alpha = 20°$ and $\beta = 16°$



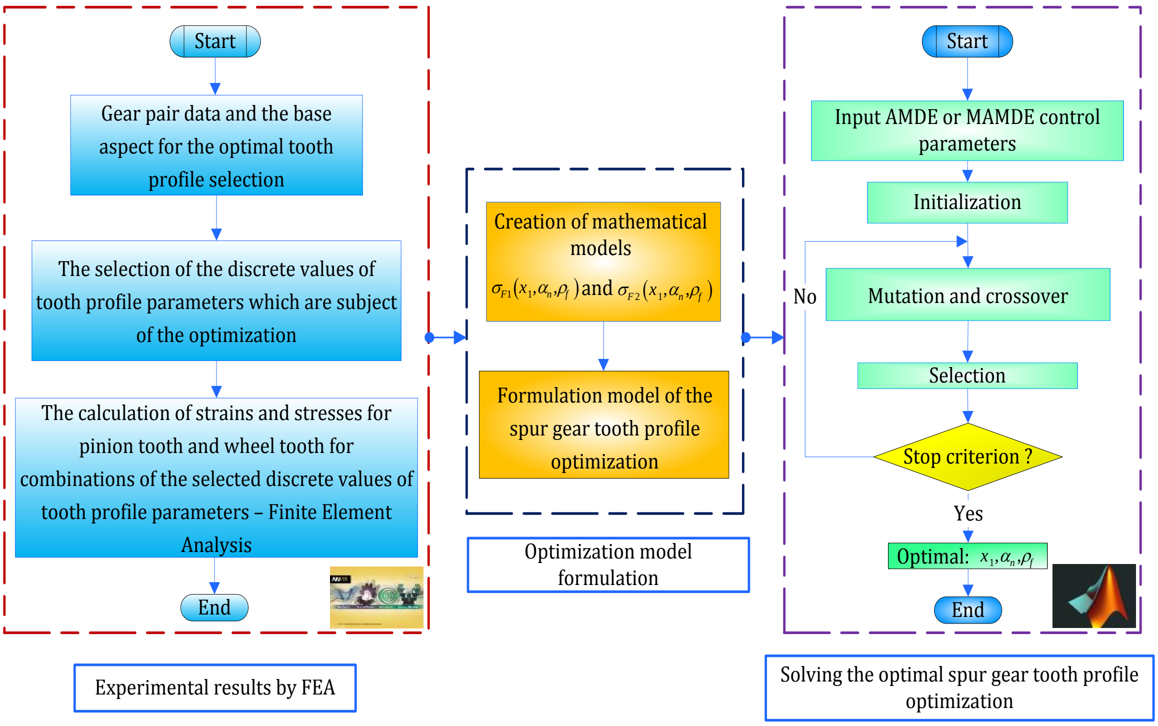
5.3. Optimal tooth profile

In the present section the AMDE and MAMDE versions are applied to select the main geometry parameters of specific cylindrical spur gear. The optimization procedure for this case is investigated for bi-objectives formulation. They are the minimizing the pinion maximum bending stresses and minimizing the maximum specific sliding. The mathematical model of the maximum bending stresses is developed using a FEA calculation. Minimal transverse contact ratio value, thickness of the tooth tip diameter and tooth interferences are considered as constraints. Design variables are the profile shift coefficient x_1, radius of root curvature ρ_f and normal pressure angle α_n (Abderazek et al. 2017).

5.3.1. Optimization procedure for gear design profile

The flowchart in Fig.5.14 displays a brief description of the developed optimization procedure for selecting the optimal tooth profile parameters of the spur gear. In Atanasovska et al. (2013) an Explicit Parametric Method (EPM) was developed to estimate the optimal spur gears tooth profile. In their study, the objective was to balance the maximum bending stresses in gear pair. The FEA was used for all necessary stress and strain calculations. The EPM was applied to optimize the profile tooth of a real spur gear in a large transport machine. The main characteristics of this gear are: number of teeth $z_1 = 20$, $z_2 = 96$; face width $b = 175$ mm; normal module $m_n = 24$ mm; normal pressure angle α_n= 20°; sum of profile shift coefficients $x_1 + x_2 = 0.5$; rotational wheel speed $n_2 = 4.1596$ rpm and wheel torque $T_2 = 1263$ kN.m.

Fig. 5.14 Flowchart of optimization procedure for gear design profile



5.3.1.1. Modeling

The authors in (Atanasovska et al., 2013) found that the exponential functions, as shown in Eqs. 5.14 and 5.15, describe how the maximum root stresses depend on the profile shift coefficient.

$$\sigma_{F1}(x_1) = a_1 e^{b_1 x_1} \tag{5.14}$$

$$\sigma_{F2}(x_1) = a_2 e^{b_2 x_1} \tag{5.15}$$

where a_1, a_2, b_1, b_2 are coefficients depending on the variables α_n and ρ_f, e is the natural base logarithm, σ_{F1}, σ_{F2} are the maximum bending tooth root stress of the pinion and the wheel respectively.

In our previous work the coefficients $a_1, a_2, b_1\ b_2$ are expressed according to variables α_n and ρ_f based on the results of numerical experiments found by Atanasovska et al. (2013). The toolbox Surface Fitting Tool of MATLAB was used to create 2D polynomials (Figs. 5.15 to 5.18) (Abderazek et al. 2017). So, the final mathematical models of the maximum bending stresses depending on the three parameters are presented as follows:

$$\sigma_{F1}(x_1,\alpha_n,\rho_f) = (2065 - 19.53\alpha_n - 96.6\rho_f)e^{(-1.437-0.00403\alpha_n+0.09107\rho_f)x_1} \tag{5.16}$$

$$\sigma_{F2}(x_1,\alpha_n,\rho_f) = (1237 - 13.39\alpha_n - 43.48\rho_f)e^{(0.3755-0.02243\alpha_n+0.06075\rho_f)x_1} \tag{5.17}$$

After modeling the maximum bending stresses in function of the profile shift coefficient, the radius of root curvature and the normal pressure angle, the mathematical model formulation of the studied spur gear is developed for bi-objective optimization. Finally, the AMDE and MAMDE algorithms are used to solve the problem.

Fig. 5.15 Polynomial 2D for $a_1(\alpha_n, \rho)$

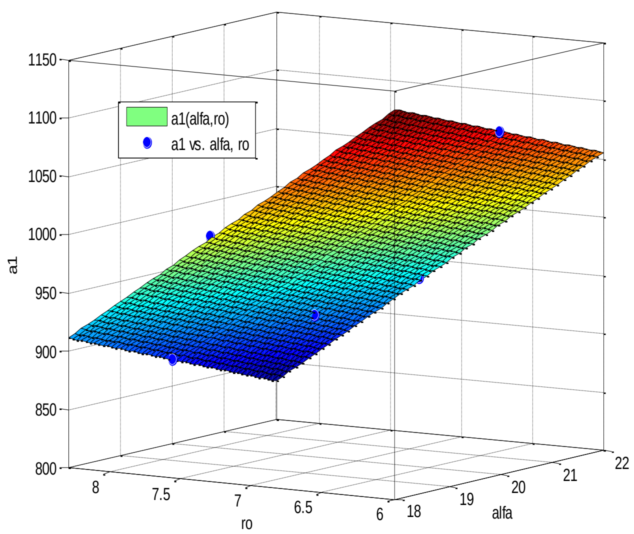


Fig. 5.16 Polynomial 2D for $a_2(\alpha_n, \rho)$

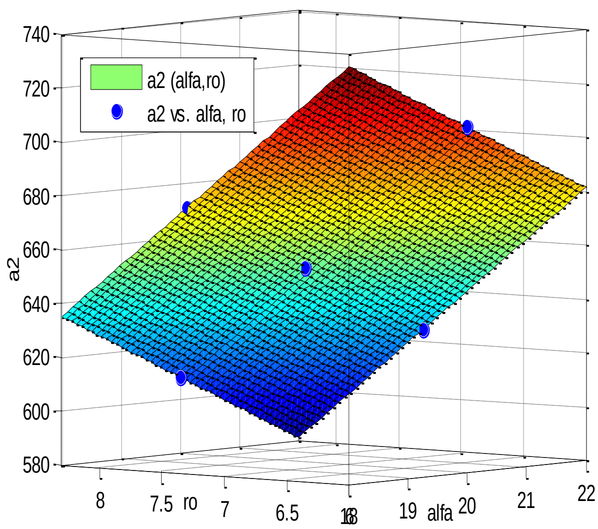


Fig. 5.17 Polynomial 2D for $b_1(\alpha_n, \rho)$

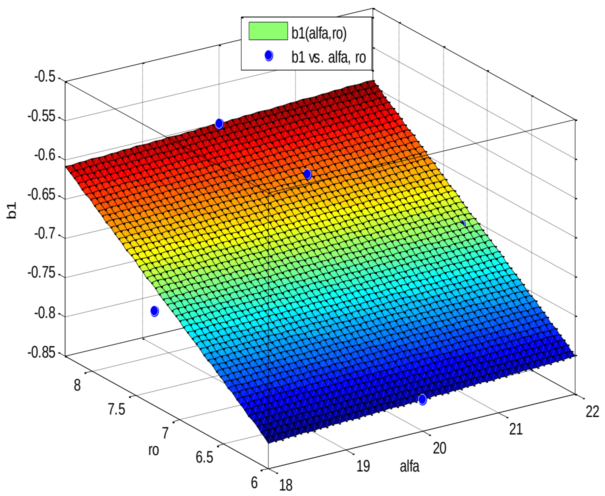
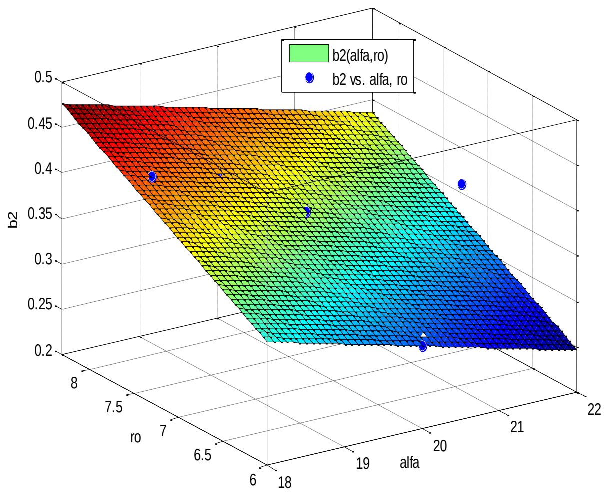


Fig. 5.18 Polynomial 2D for $b_2(\alpha_n, \rho)$



5.3.1.2. Formulation problem of gear design profile

Objective functions: the studied case is a real gear pair in a large transport machine (Atanasovska et al. 2013). The optimal tooth profile problem is formulated as a bi-objective optimization problem, among the objectives are: minimizing the pinion maximum bending stress and minimizing the maximum specific sliding. So, the first objective can mathematically expressed as following:

$$\text{Minimize:} \quad f_1(x) = (2065 - 19.53\alpha_n - 96.6\rho_f)e^{(-1.437-0.00403\alpha_n+0.09107\rho_f)x_1} \tag{5.18}$$

The second objective can expressed as:

$$\text{Minimize:} \quad f_2(x) = \frac{\tan\alpha_{at1} - \tan\alpha_t'}{(1+u)\tan\alpha_t' - \tan\alpha_{at1}}(u+1) \tag{5.19}$$

where $x = \{x_1, \alpha_n, \rho_f\}$ is the vector of design variables.

For solving this bi-objective problem the weighted sum method is used. The basic idea of this method is to convert the multi objective problem to the single one by using the weighting factors. For the present case, the weighting factors are taken as: $\beta_1 = 0.9$, $\beta_2 = 0.1$. Thus, the objective function can be formulated as:

$$\text{Minimize:} \quad f(x) = \beta_1 \times f_1(x_1, \alpha_n, \rho_f) + \beta_2 \times f_2(x_1, \alpha_n) \tag{5.20}$$

Constraints: the objective functions are subjected to seven constraints. The five first constraints taking in consideration the kinematic condition and they are the same as in the first application g_1, g_2, g_3, g_4 and g_5.

- To provide equally strong teeth on the pinion and the wheel, their maximum bending stresses should be balanced as:

$$g_6(x) = |\sigma_{F2} - \sigma_{F1}| < 10^{-5} \tag{5.21}$$

- To maximize the wear resistance of the gear pair the maximum specific sliding coefficients must be equal at extremes of contact path:

$$g_7(x) = |\gamma_{2\max} - \gamma_{1\max}| < 10^{-5} \tag{5.22}$$

Design variables: The final optimization model of the spur gear design profile is developed for three mixed design variables. The profile shift x_1 and the radius of root curvature ρ_f are continuous, and the pressure angle α_n is discrete. The upper and the lower variables limits are: $0.5 \le x_1 \le 0.8, 6 \le \rho_f \le 8.3$ and $\alpha_n(°) \in (18, 19, 20, 21, 22)$.

5.3.2. Results and discussions

The parameters used by the both AMDE and MAMDE for optimization search process are: NP = 50, FEs = 20,000, the scaling factors and the crossover factor are kept the same as in chapter 4 ($F_1 =$ 0.2, $F_2 =$ 0.3, Cr =0.4, τ_1 =0.5, τ_2 =0.8 and τ_3 =0.5). The statistical results of both our algorithms for the spur gear tooth profile problem are summarized in Table 5.4. In addition, the values of objective functions, design variables and constraints obtained are presented in Table 5.5.

The best solution reported by the proposed algorithms is (x_1= 0.401871, ρ_f = 6.281469 mm and α_n= 21°), corresponding to the objective functions values $f_1 = 765.89045\ MPa$ and $f_2 = 1.00916$. The constraints violation are [-0.01602839, -2.67250602, -8.885974, -42.414224, -605.257518, -4.826594E-14, -8.936842E-12]. Furthermore, from the obtained results it can be clearly seen that the design variables values really lead to a perfect balancing in both parameters, the tooth root stresses as well as the maximum specific sliding coefficients for the studied gear pair. For more detail, the evolution of specific sliding distribution along the contact path is plotted in Fig. 5.19. From the figure it can clearly observed that specific sliding at the tip and the root of the teeth is well balanced, such as at the tip ($\gamma_{1\max\,(E)} = \gamma_{2\max\,(A)}$ = 0.5022) and at the root ($\gamma_{1\max\,(E)} = \gamma_{2\max\,(A)}$ = 1.0091). Consequently, it may be assumed that the mating gear teeth flanks make sure a regulated resistance to bending stresses and to wear, leading to an extended gear service life.

From the statistical results, it's clear that the MAMDE version is more robust than the AMDE in solving this problem with lower standard deviation 3.4973E-09. In addition, the AMDE version has difficulty in finding the feasible solutions. In other words, it is not stable to give feasible solution for the 50 runs (the feasible rate is 29/50). This problem is due to the penalty mechanism used to deal with constraints functions and the local search-ability of the AMDE that still needs to be improved. It may be noted that for AMDE the statistical results are calculated based on the 29 feasible runs.

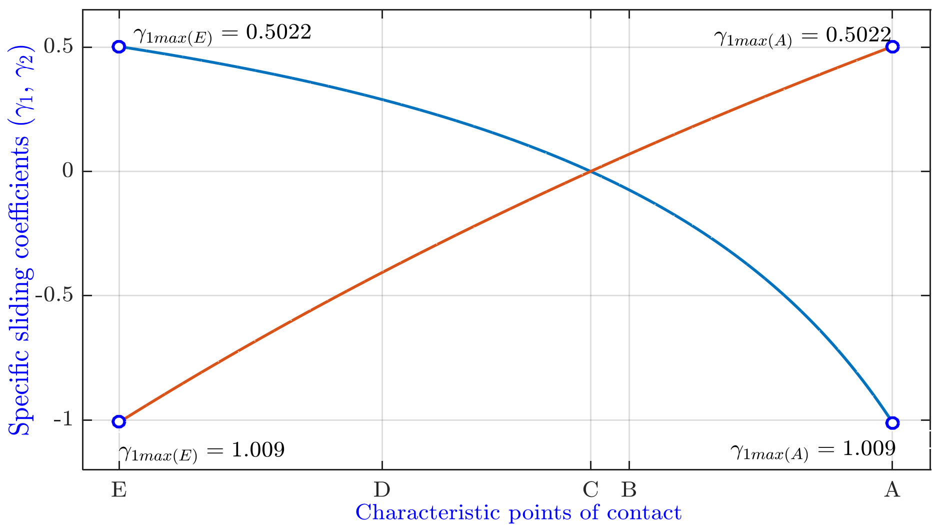
Table 5.4 Statistical results of MAMDE and AMDE for spur gear tooth profile problem

Algorithm	Best	Mean	Worst	SD	FEs
MAMDE	689.4023280159668	689.4023280167306	689.4023280397285	**3.4973E-09**	20,000
AMDE	689.4023280159663	689.4023316878637	689.4024344884042	1.9771E-05	20,000

Table 5.5 Values of objective functions, design variables and constraints for spur gear tooth profile problem

Variables	Algorithms	
	AMDE	MAMDE
x_1	0.401985712227	0.401985712227
ρ_f	6.281469068614	6.281469068613
α_n	21.00000000000	21.00000000000
g_1	-0.016028397432	-0.016028397432
g_2	-2.672506026580	-2.672506026580
g_3	-8.885974893161	-8.885974893161
g_4	-42.41422494407	-42.41422494407
g_5	-605.2575185630	-605.2575185630
g_6	-4.82659458E-14	-4.82659458E-14
g_7	-8.93684298E-12	-8.93684298E-12
σ_{F1}	765.8904568384	765.8904568384
σ_{F2}	765.8904568385	765.8904568384
$\gamma_{1\max}$	1.009168614432	1.009168614432
$\gamma_{2\max}$	1.009169614432	1.009169614432
$f_{\min}$	689.4023280160	689.4023280160

Fig. 5.19 Evolution of specific sliding distribution along the contact path for the case study



5.3.2.1. Finite element analysis using ANSYS

The optimum design variables obtained through the optimization procedure using the both algorithms AMDE and MAMDE of the spur gear are used for preparation of a CAD model. Then the stress analysis using finite element analysis is performed by using ANSYS APDL in order to check the results found in the previous section. Only the characteristic points of the path of contact with maximum load values are taken into consideration during the FEA calculations. As we mentioned previously in the general introduction, the developing of the CAD model and the calculation by using FEA for optimization verification is done by Dr. Ivana Atanasovska from the Mathematical Institute of the Serbian Academy of Sciences and Arts, (Atanasovska et al., 2010; Atanasovska et al., 2013).

Figures 5.20 and 5.21 show the details of discrete models (FE meshes) developed for a particular gear pair with the standard tooth profile shift coefficients in accordance with the results of the previous performed optimization procedure. The obtained Von Mises stress distribution of the spur gear train is presented in Figs. 5.22 and 5.23. The maximum bending stress (Von Mises) calculated at the pinion root is: $\sigma_{VonMises}(root - \max) =$ 765.30665 *MPa*, and their-of the wheel is: $\sigma_{VonMises}(root - \max) =$ 765.50232 *MPa*. From the results, it could be observed that maximum bending stresses are not really equal, and this is due to the small deviations between the values of the optimal shift coefficients of the tooth profiles of the pinion and the wheel obtained through the optimization procedure using the AMDE and MAMDE algorithms and the nearest standard values of these coefficients selected for FEA.

Fig. 5.20 Two-dimensional FEM model of the pinion – single meshed period

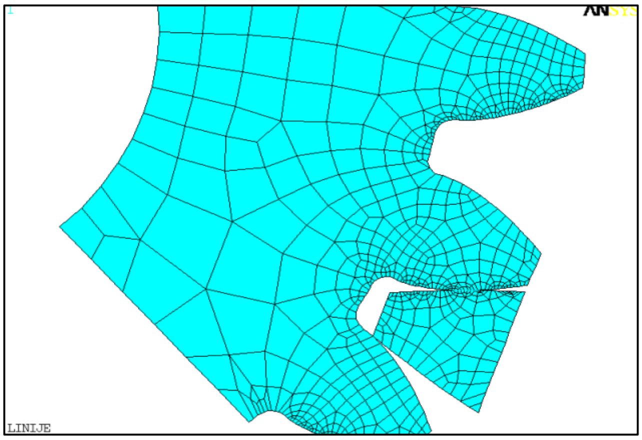


Fig. 5.21 Two-dimensional FEM model of the wheel– single meshed period

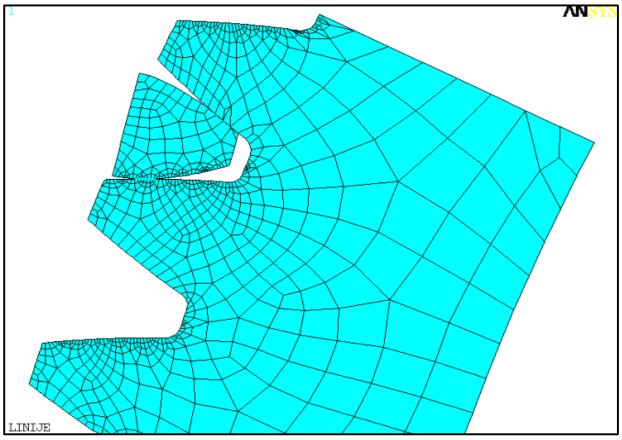
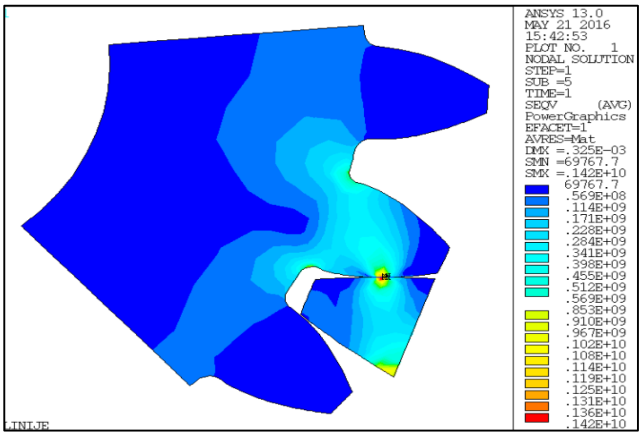


Fig. 5.22 Equivalent stress results for the pinion



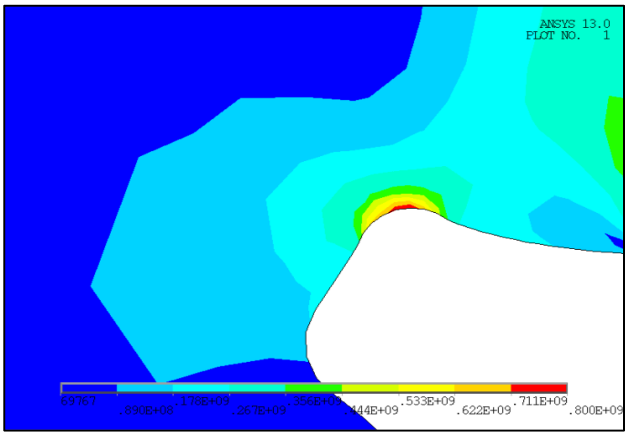
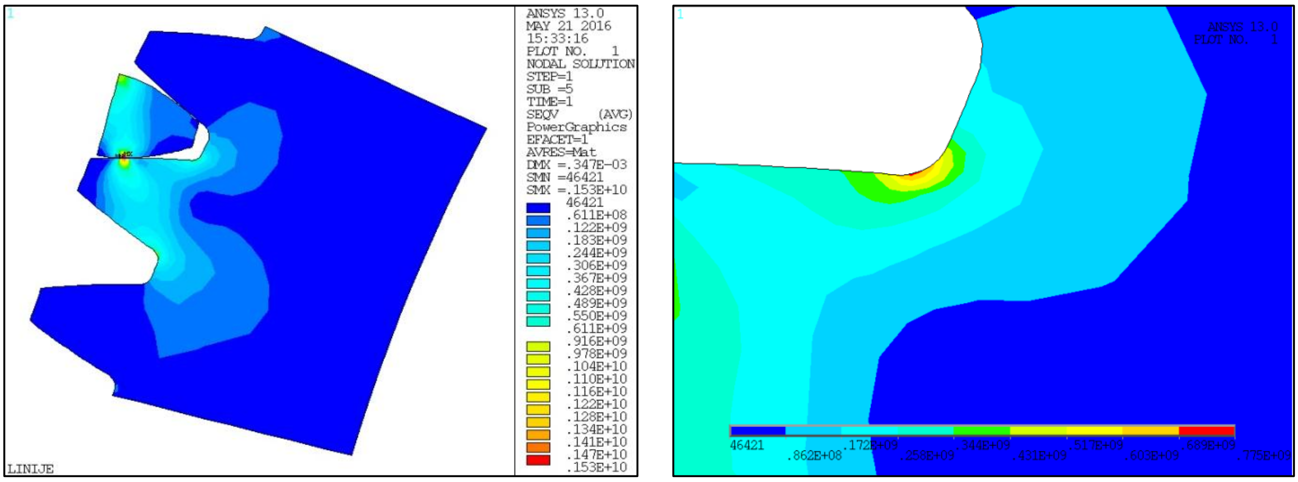


Fig. 5.23 Equivalent stress results for the wheel



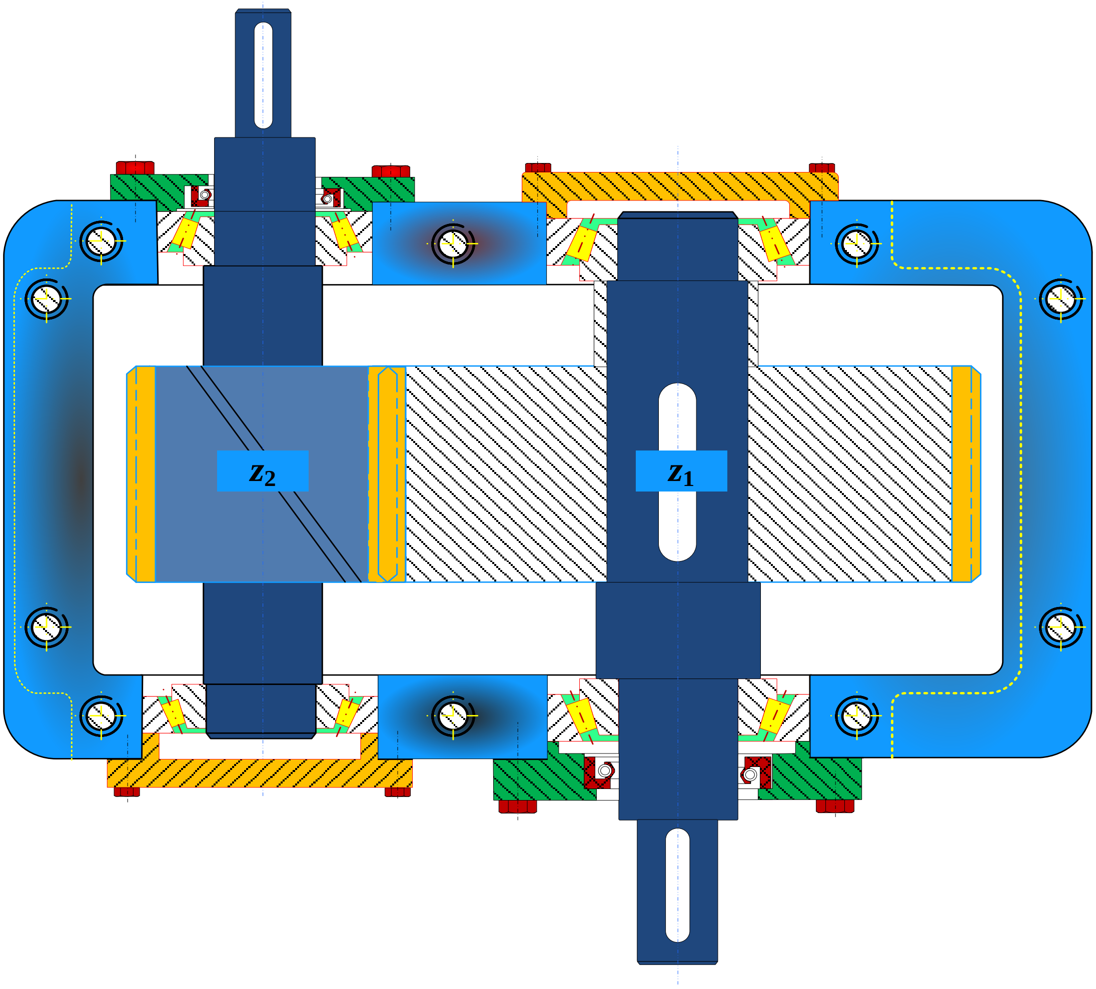
5.4. Design of gear train

In this part we apply the developed algorithms to optimize design of a single stage cylindrical speed reducer (Fig. 5.24), where the objectives are to minimize the gear masse, minimize sliding velocity of pinion and maximize gear efficiency. The mathematical model of the optimization problem is developed including nine mixed variables among them: the normal module m_n, number of teeth z_1 and z_2, helix angle β, normal pressure angle α_n, face widths b_1 and b_2 and profile shift coefficients x_1 and x_2. In addition, the tooth root strength, surface durability (pitting) and wear resistance are constrained to remain within the prescribed values, and other design criteria such as: minimal transverse contact ratio, minimal helical contact ratio, nominal transmission ratio, operating gear interferences, are also considered. The type and the range of each design variable are as following:

- m_n – discrete real values in the range of [1,…, 50] (ISO 54:1996);
- z_1 –integer values in the range of [16,…,50];
- z_2 –integer values in the range of [20 ,…,500];
- β – helix angle measured at the pitch diameters, integer values in the range of [10,…, 20];
- α_n – discrete real value in the range of [14,…, 20];
- b_1 and b_2 –real values in the range of the range of [10,…,1000];
- x_1 and x_2 – real values in the range of [-0.5,…,1].

As we mentioned in the second chapter (see last paragraph sub section 2.4.1.2) the geometric calculation of gear pair shifted by the S_0-gearing or the S-gearing systems is based on DIN 3960:1987 recommendation. For the resistance verification of gear system, we fallowed the simplified methodology proposed by Henriot (2007) in his practical handbook of gear design. In addition, for bending strength and contact stress calculations we based on recommendations of DIN 3990:1987, ISO 6336-2:2006, ISO 6336-3:2006, ISO 9085:2002, AGMA 2001-D04:2004.

Fig. 5.24 Schematic view of single stage helical speed reducer



5.4.1. Objective functions

Objective 1: A reduced volume of machine element means metal economy and cheaper production (Buiga and Tudose, 2014). Moreover, the gear weight presents an important portion of total weight of the speed reducer. Taking these aspects into account, the minimization of gear mass is the first objective. It should be noted that the gear considered as solid structure and its volume is approximated by the volume of a cylinder with the tip diameter as outer diameter, mathematically can be described as follows:

$$\text{Minimize:} \quad f_1(x) = \frac{\pi \times m_n^2}{4}\left[b_1 \times \rho_1\left(\frac{z_1}{\cos\beta} + 2h_{a1}^* + 2x_1\right)^2 + b_2 \times \rho_2\left(\frac{z_2}{\cos\beta} + 2h_{a2}^* + 2x_2\right)^2\right] \tag{5.23}$$

where $x = \{m_n, z_1, z_2, \beta, \alpha_n, b_1, b_2, x_1, x_2\}$ is the vector of design variables, ρ_1 is the density of the pinion materiel, and ρ_2 is the density of the wheel materiel.

Objective 2: The sliding velocity is important for power loss (efficiency, heating) and scuffing strain (Heinz and Ralf, 2016). In order to maximize the service life and wear resistance of the gear pair, the maximum sliding velocity of pinion should be minimized. So, the second objective function can be expressed as:

$$\text{Minimize: } f_2(x) = \omega_1 \times \sqrt{r_{a1}^2 - r_{b1}^2} - \omega_2 \times \left(\sin\left(\alpha_t^{'}\right) - \sqrt{r_{a2}^2 - r_{b2}^2} \right) \tag{5.24}$$

where $\omega_{1,2}$ are the angular velocities of pinion and wheel, r_{a1} and r_{a2} denotes respectively the tip radii of pinion and wheel.

Objective 3: The gear mesh efficiency calculation is carried out according to Henriot (2007), and their mathematical formulation is expressed as:

$$\text{Maximize : } f_3(x) = \left(1 - f_m \left(\frac{2}{d_1} + \frac{2}{d_2} \right) \times \left(\frac{g_f^2 + g_a^2}{2(g_f + g_a)\cos\left(\alpha_t^{'}\right)} \right) \right) \tag{5.25}$$

where g_f is the approach path length, g_a is the recess path length, f_m is the mean value of the friction coefficient taken in range [0.04, 0.07], d_1, d_2 are the reference diameter of pinion and wheel, respectively.

5.4.2. Design constraints

The objective functions listed above are subjected to the following constraints:

5.4.2.1. Calculation of Hertzian stress

For mechanical resistance verifications (bending and contact stress) of the gear pair, we use the simplified methodology proposed by Henriot (2007). The method is based on the calculations of permissible transmittable power for contact and bending stresses. Thus, first condition is the minimum transmittable power for the permissible Hertzian stress. Mathematically the constraint is expressed as:

$$g_1(x) = -\frac{C_{H1} \times C_{H2} \times C_{H3} \times C_{H4} \times C_{H5} \times C_{H6}}{K_A \times S_{H\min}^2} \times Z_{NT}^2 + P_{\min} \leq 0 \tag{5.26}$$

where Z_{NT} is the life factor, S_{Hmin} is the minimum safety factor for surface durability, P_{min} is the transmitted power, and K_A is the application factor which takes into account the operating characteristics of the driving and driven equipment. $C_{H(i)}, i = \{1,2,\dots,6\}$ are factors that take in consideration, the geometry, dynamic and operational effects of a gear in order to estimate a real value of contact stress. Each factor is calculated as following:

- Factor C_{H1}:

$$C_{H1} = 52.4 \times 10^{-3} n_1 \frac{u}{u+1} \tag{5.27}$$

where n_1 is the pinion rotational speed (rpm).

- Factor C_{H2} (geometric factor):

$$C_{H2} = \frac{1}{\left(Z_H \times Z_\varepsilon \times Z_\beta\right)^2} \tag{5.28}$$

where the zone factor, Z_H, take into account the pressure angle and helix angle for the calculation of the pressure in pitch point, Z_ε is the contact ratio factor (pitting) which accounts for the influence of the transverse contact and overlap ratios on the surface load capacity of cylindrical gears, and Z_β is the helix angle factor (pitting). The three factors Z_H, Z_ε and Z_β are calculated according ISO 9085:2002.

- Factor C_{H3} (velocity factor):

$$C_{H3} = \frac{Z_v^2}{K_v} \tag{5.29}$$

where Z_v is the velocity factor, the dynamic factor, K_V accounts for internally generated dynamic loads due to vibrations of pinion and wheel against each other and is calculated using the method C of DIN 3960-1:1987.

- Factor C_{H4} (Distribution factor):

$$C_{H4} = \left(\frac{b_1 \times d_1^2}{K_{H\beta} \times K_{H\alpha}}\right) \times 10^{-6} \tag{5.30}$$

where d_1 denotes the diameter of the reference circle, $K_{H\beta}$ is the face load distribution factor which calculated according AGMA 2001-D04:2004, and $K_{H\alpha}$ is the transverse load distribution factor.

- Factor C_{H5} (material factor):

$$C_{H5} = \left(\frac{\sigma_{H\lim}}{Z_E}\right)^2 \times Z_W^2 \tag{5.31}$$

where σ_{Hlim} is the allowable stress numbers (contact), Z_E is the elasticity factor which take into account the influences of the material properties, and Z_W is the work hardening factor.

- Factor C_{H6}:

$$C_{H6} = Z_L^2 \times Z_R^2 \times Z_W^2 \tag{5.32}$$

where Z_L is the lubrication factor, Z_R is the roughness.

For more details about the different factors reader can refer to the standard (ISO 6336-2:2006 method B).

5.4.2.2. Calculation of tooth bending strength

The second constraint is the minimum transmittable power for the permissible tooth bending stress. Mathematically the constraint is expressed as following:

$$g_2(x) = -\frac{C_{F1} \times C_{F2} \times C_{F3} \times C_{F4} \times C_{F5} \times C_{F6}}{K_A \times S_{F\min}} \times Y_{NT} + P \leq 0 \tag{5.33}$$

where Y_{NT} is the life factor, S_{Fmin} is the minimum safety factor for tooth breakage, and $C_{F(i)}, i = \{1,2, \dots ,6\}$ are the factors that take in consideration, the geometry, dynamic and operational effects of a gear in order to estimate a real value of bending stress. Each factor is calculated as following:

- Factor C_{F1}:

$$C_{F1} = 52.4 \times 10^{-3} \times n_1 \times z_1 \times m_n^2 \tag{5.34}$$

- Factor C_{F2}:

$$C_{F2} = \frac{1}{Y_\beta \times Y_\varepsilon \times \cos\beta} \tag{5.35}$$

where Y_β is the helix angle factor (tooth root) and Y_ε is the contact ratio factor (tooth root).

- Factor C_{F3}:

$$C_{F3} = \frac{10^{-3} \times b_1}{K_{F\beta} \times K_{F\alpha}} \tag{5.36}$$

where $K_{F\alpha}$ is the transverse load factor (root stress), and $K_{F\beta}$ is the face load factor (root stress).

- Factor C_{F4}:

$$C_{F4} = (Y_{\delta relT} \times Y_{RrelT}) \times Y_{\mathrm{x}} \tag{5.37}$$

where $Y_{\delta relT}$ is the relative notch sensitivity factor, Y_{RrelT} is the surface factor, and Y_X is the size factor (tooth root).

- Factor C_{F5}:

$$C_{F5} = \sigma_{F\lim} \times Y_{ST} \tag{5.38}$$

where σ_{Flim} is the allowable stress number (bending), and $Y_{ST}$is the stress correction factor.

- Factor C_{F6}:

$$C_{F6} = \frac{1}{Y_{Fa} \times Y_{Sa}} \tag{5.39}$$

where Y_{Fa} is the tooth form factor, and Y_{Sa} is the stress correction factor. They are calculated according the method B of the ISO: 6336-2006 standard.

5.4.2.3. Constructive and geometrical constraints

- The transverse contact ratio ε_α should be greater than a certain imposed limit value $\varepsilon_{\alpha\lim}$:

$$g_3(x) = \varepsilon_{\alpha\lim} - \varepsilon_\alpha \le 0 \tag{5.40}$$

- Also the axial overlap ratio ε_β should be greater than a certain imposed limit value $\varepsilon_{\beta\lim}$:

$$g_4(x) = \varepsilon_{\beta\lim} - \varepsilon_\beta \le 0 \tag{5.41}$$

- Relative error of the actual speed ratio has to be between -2.5 % and +2.5%.

$$g_5(x) = \left| u_{th} - \frac{z_2}{z_1} \right| - 0.025 \times u_{th} \le 0 \tag{5.42}$$

- To ensure a regular load distribution, the ratio b / m_n should be in the range [8, 16]:

$$g_6(x) = 8 - \frac{b}{m_n} \le 0 \tag{5.43}$$

$$g_7(x) = \frac{b}{m_n} - 16 \le 0 \tag{5.44}$$

- The profile shift should be in such range that the pinion and the wheel tooth thickness at top does not decrease under a certain limit value:

$$g_8(x) = c_{Sa\lim} \times m_n - s_{an1} \le 0 \tag{5.45}$$

$$g_9(x) = c_{Sa\lim} \times m_n - s_{an2} \le 0 \tag{5.46}$$

- To avoid the operating interference :

$$g_{10}(x) = d_{a2} - 2 \times \sqrt{r_{b2}^{\,2} + \left(a' \sin \alpha_t'\right)^2} \le 0 \tag{5.47}$$

$$g_{11}(x) = d_{a1} - 2 \times \sqrt{r_{b1}^{\,2} + \left(a' \sin \alpha_t'\right)^2} \le 0 \tag{5.48}$$

- The profile shift coefficient should be in such range that the undercutting of the pinion teeth doesn't get worse.

$$g_{12}(x) = \frac{z_{\min} - z_1}{z_{\min}} h_{an1}^* - x_1 \le 0 \tag{5.49}$$

$$g_{13}(x) = \frac{z_{\min} - z_2}{z_{\min}} h_{an2}^* - x_2 \le 0 \tag{5.50}$$

- The sum of profile shifts should be in the range of [-0.5, 1] for general mechanical using.

$$g_{14}(x) = -\left(0.5 + x_1 + x_2\right) \le 0 \tag{5.51}$$

$$g_{15}(x) = x_1 + x_2 - 1 \leq 0 \tag{5.52}$$

- The maximum sliding velocity should be equal at extremes of contact path.

$$g_{16}(x) = \left| v_{gE} - v_{gB} \right| \leq 10^{-3} \tag{5.53}$$

- The maximum efficiency should be greater than 97%

$$g_{17}(x) = -\eta_{gear} + 0.97 \leq 0 \tag{5.54}$$

where is the gear mesh efficiency (Eq. 5.25)

Through the optimization procedure of a single stage cylindrical speed reducer and for each iteration the Newton-Raphson method is used to solve nonlinear equation in order to determine the values of the working pressure (α'_n or α'_t) angle and working center distance. As we know, the two parameters depends on the number of teeth on the pinion z_1, the number of teeth on the wheel z_2, profile shift coefficient of the pinion x_1 and profile shift coefficient of the wheel x_2. Also, this numerical method is performed to determine the form factor Y_{Fa} and the stress correction factor Y_{Sa}. Generally, this method converges to the best solution after five iterations for the two cases. Solving of these equations is performed in separate procedures, and the values obtained for each angle are dynamically transferred in the main program.

The life factors Z_{NT} and Y_{NT} are mathematically expressed in function of the loading cycles N_L, and the material type for the gear pair using polynomial interpolation. The data for these factors are extracted from flowcharts that presented in ISO 6336: 2006 parts 2 and 3. The toolbox Fitting Tool of MATLAB is used. In addition, separate procedures are created to calculate the dynamic factor K_V using the method C of DIN 3960-1:1987, the hardness ratio, the form factor, and the stress correction factor. For each iteration the values obtained from each procedure are dynamically transferred to the main program.

5.4.3. A practical design examples

AMDE and MAMDE are applied in order to design two gear trains one is helical and the other is spur. The input variables such as transmitted power, input speed, total gear ratios, and other data are given in Table 5.6. As we mentioned previously the objectives are to obtain a minimum mass of gear, minimum sliding velocity and maximum efficiency for a service life of 20,000 h, an input speed of 2950 *rpm* and 440.8 *rpm* and a total transmission ratio of 4 and 3 respectively for each case study. The gear of the speed reducer, for the two cases, should be based on an ISO 53:1998 basic rack profile ($h^*_{aP} = 1, h^*_{fP} = 1.25, \rho^*_{fP} = 0.38$ and $C^*_P = 0.25$).

Table 5.6 Gear design parameters used in the optimization process

Parameters	Symbol	Unit	Case I: Helical gear		Case II: Spur gear	
			Pinion	**Wheel**	**Pinion**	**Wheel**
Type of gear	-	-	Helical		Spur	
Mesh type	-	-	External		External	
Power	P	kW	12.500		75.0000	
Speed	n	1/min	2950.0	2950.0/4	440.800	440.800/3
Required transmission ratio	u_{th}	-	4		3	
Required service life	L_h	h	20,000		20,000	
Reference profile	-		A	A	A	A
Precision class	Q	-	6	6	6	6
Application factor	K_A	-	1.25		1.25	
Material	-	-	31CrMoV9	42CrMo4(3)	18CrNiMo7-6	18CrNiMo7-6
Material type	-	-	NT(nitr.)(*)	NT(nitr.)(*)	V(*)	V(*)
Allowable-stress number	$\sigma_{H\,\lim}$	N/mm²	1250.00	1000.00	1500	1500
Nominal-stress number	$\sigma_{F\,\lim}$	N/mm²	425.00	370.00	430	430
Brinell hardness	HB	-	601	504	642	642
Young's modulus	E	N/mm²	206000	206000	206000	206000
Poisson's ratio	υ	-	0.30	0.30	0.30	0.30
Material density	ρ	kg/m³	7830.00	7830.00	7830.00	7830.00
Mean-peak-to-valley roughness	R_z	μm	20.00	20.00	4.8	4.8
Lubrication type	-	-	Oil: Mobilgear 600 XP 150		Oil: GEM 1-220 N Klüberoil	
Kinematic-viscosity of the oil at 40°C	ν_{40}	mm²/s	150.00		220.00	
Stress-correction factor	Y_{ST}	-	2.00		2.00	
Sensitivity factor	$Y_{\delta relT}$	-	1.00		0.997	
Surface factor	Y_{RrelT}	-	0.9		0.957	
Size factor	Y_x	-	0.85		0.99	
Limit value	$c_{Sa\,\lim}$	-	0.4	0.4	0.35	0.35
Limit value	$\varepsilon_{\alpha}\lim$	-	1.3		1.4	
Limit value	$\varepsilon_{\alpha}\lim$	-	0.8		0	

(*): See ISO 6336-1:2006, Table 2, for an explanation of the abbreviations used for the material type.

5.4.3.1. Results and discussions

The parameters used by AMDE and MAMDE for optimization search process are: NP = 80, FEs = 150,000, the scaling factors and the crossover factor are kept the same as in chapter 4 ($\boldsymbol{F_1}$ =0.2, $\boldsymbol{F_2}$ =0.3, $\boldsymbol{Cr}$ = 0.4, $\boldsymbol{\tau_1}$ = 0.5, $\boldsymbol{\tau_2}$ =0.8 and $\boldsymbol{\tau_3}$ = 0.5). The convergence graph for each case studied using both algorithms is presented in Figures 5.25 and 5.27, respectively.

As it can be observed from figures, AMDE and MAMDE converged for the global optimal solution with total satisfaction of the constraints violation for the two considered cases (as shown in Figures 5.26 and 5.28). Regarding the convergence speed (case I), for the first iterations the AMDE is slightly faster than MAMDE (as shown in zoom part 1 in Fig. 5.25), but after that MAMDE is much faster and converged to best solution before AMDE (zoom parts 2 and 3 in Fig. 5.25). For the spur gear, it is easy to see from the three zoom parts in Fig. 5.27 that AMDE is faster than MAMDE.

The computational time for each case and each algorithm is important due that for each iteration we should solve three nonlinear equations using Newton-Raphson method in order to determine α_t', Y_{Fa} and Y_{Sa}.

Fig. 5.25 Convergence graph of AMDE and MAMDE for the helical speed reducer (case I)

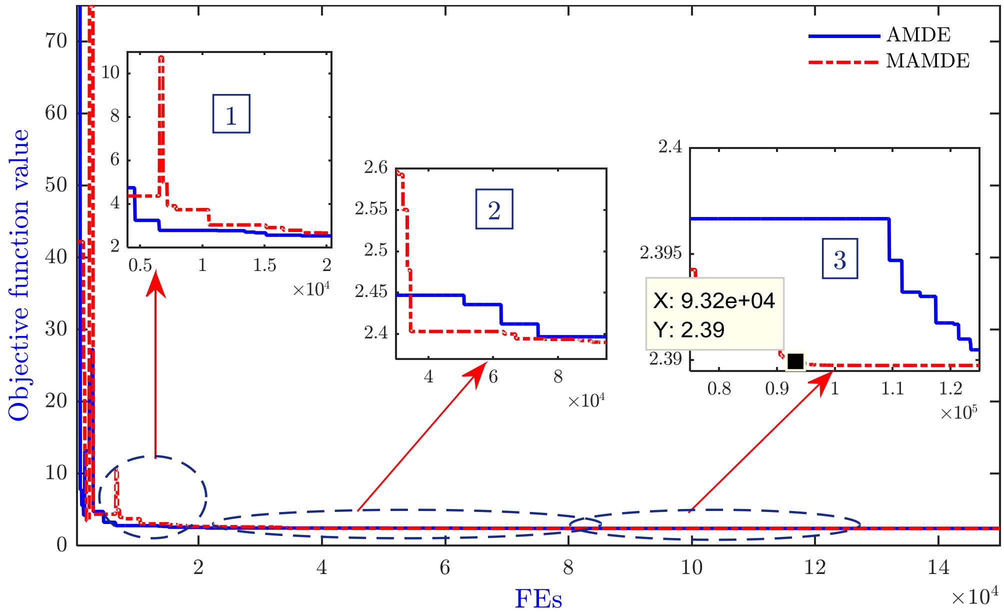


Fig. 5.26 Constraints violation for AMDE and MAMDE (speed reducer: case I)

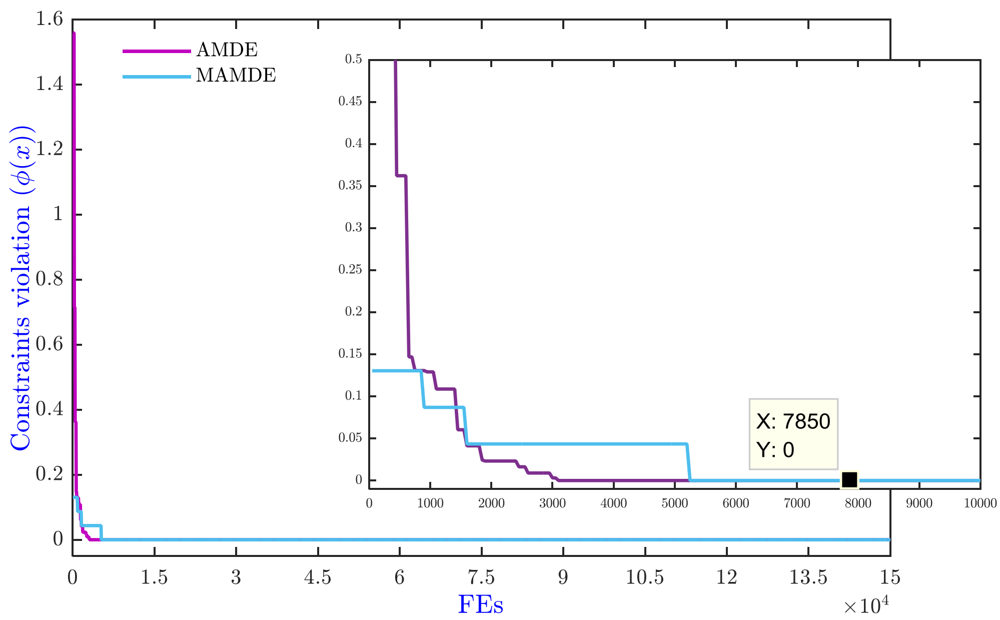


Fig. 5.27 Convergence graph of AMDE and MAMDE for the spur speed reducer (case II)

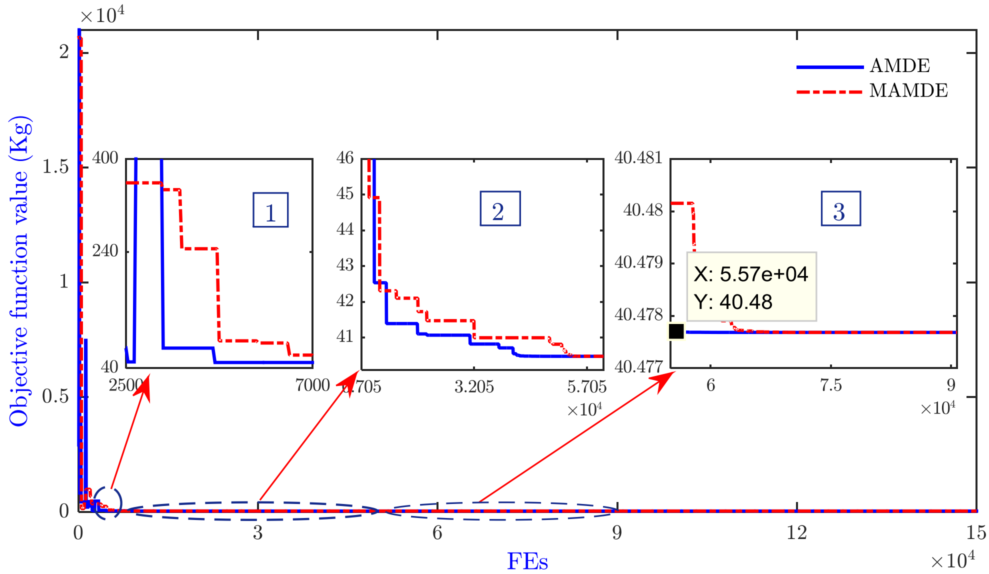
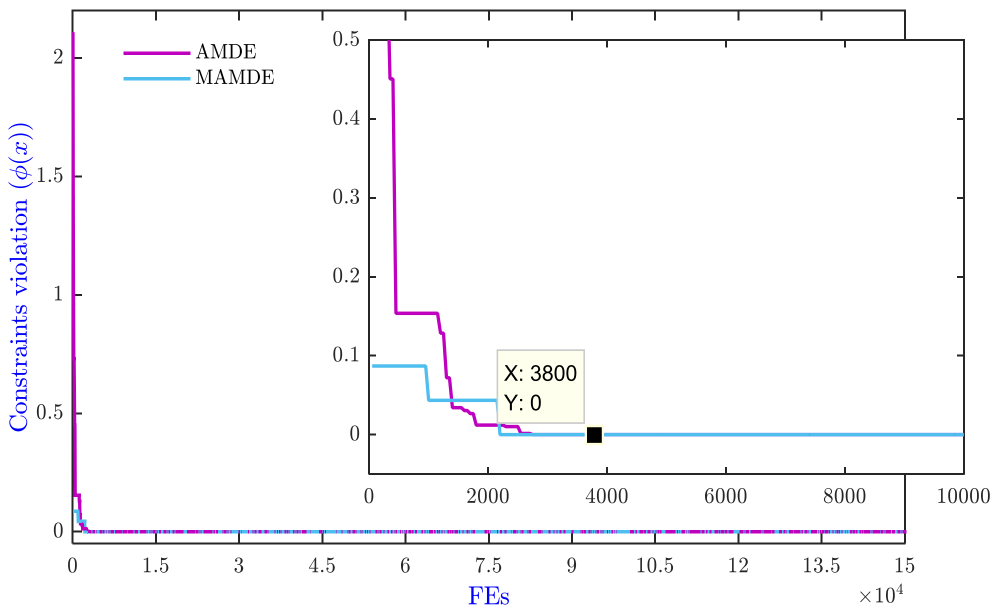


Fig. 5.28 Constraints violation for AMDE and MAMDE (speed reducer: case II)



The optimal results obtained by using the developed algorithms are compared with those provided by Traditional Design (TD) method given in (Jelaska, 2012). The main geometrical parameters of both cases for the three methods are presented in Table 5.7. For both cases, the optimal solutions offered by AMDE and MAMDE are the same. It should be noted that the calculations of gear geometry and mechanical resistance for the two cases are validated by using the commercial software KISSsoft (KISSsoft, 2013). To show a good comparison between the results given by the three methods, we choose to present graphically the first fitness value (the weight of the gears) and two optimal variables which are pinion teeth number and face width, Figures 5.29, 5.30 and 5.31, respectively.

For the first objective, from Fig. 5.29 and Table 5.8 it can be clearly observed that our algorithms offer a mass reduction of 33.19 % for the helical speed reducer (mass = 2.38975 kg) and 38.55 % for the spur speed reducer (mass = 40.47768 kg) comparing with the results given by traditional method (mass = 3.577 kg for the first case and mass = 65.877 kg for the second case). These reductions in weights of the gears will directly influence in the manufacturing costs which will be significantly diminished. For the other parameters such as the pinion teeth and face width for both cases, our algorithms delivered small values compared with TD method (as shown in Fig.5.30 and 5.31). Where $z_1 = 21$ for the two cases using our variants and $z_1 = 27$ and 25 using TD method for the first speed reducer and the second respectively. For the module all compared methods achieved the same value for the helical and the spur speed reducers.

Considering the mechanical resistance it is clear from the Table 5.7 that the optimal results obtained by AMDE and MAMDE variants, satisfied the allowable values of transmittable power for permissible contact stress and permissible bending stress (P_{HP} = 12.79kW and P_{FP} = 23.9kW) for the first case, and (P_{HP} = 90.06 kW and P_{FP} = 98.78 kW) for the second case. For the sliding velocity, $v_{g1} = v_{g2} =$ 1.37 *m/s* for the helical gear and $v_{g1} = v_{g2} = 0.80$ *m/* for the spur gear.

Table 5.7 Comparison of the optimal results for the two cases (MADE, MAMDE and TD)

Variables	Unit	Case I: Helical gear			Case II: Spur gear		
		AMDE	MAMDE	TD	AMDE	MAMDE	TD
m_n	(mm)	1.5000	1.5000	1.5000	6.000	6.000	6.000
z_1	-	21	21	27	21	21	25
z_2	-	82	82	104	62	62	76
β	(°)	19	19	20	0	0	0
α_n	(°)	20	20	14	20	20	20
b_1	(mm)	20.3220	20.3220	21.0000	38.0889	38.0889	44.00
b_2	(mm)	20.3220	20.3220	20.0000	38.0889	38.0889	44.00
x_1	-	0.11352	0.11352	0.3255	0.31925	0.31925	0.2485
x_2	-	0.085192	0.085192	0.0739	0.65718	0.65718	-0.2485
v_{g1}	(m/s)	1.3787	1.3787	1.622	0.80285	0.80285	1.056
v_{g2}	(m/s)	1.3797	1.3797	1.157	0.80185	0.80185	0.757
η_{gear}	-	98.412	98.412	98.869	99.168	99.168	99.217
ε_α	-	1.52835	1.52835	1.571	1.4660	1.4660	1.669
ε_β	-	1.40400	1.40400	1.027	0	0	0
P_{HP}	(kW)	12.79	12.500	17.46	90.06	90.06	147.09
P_{FP}	(kW)	23.94	23.87304	27.60	98.78	98.78	132.29
Mass	(kg)	2.38975	2.38975	3.577	40.47768	40.47768	65.877

Fig. 5.29 Optimized gears mass (AMDE, MAMDE and TD)

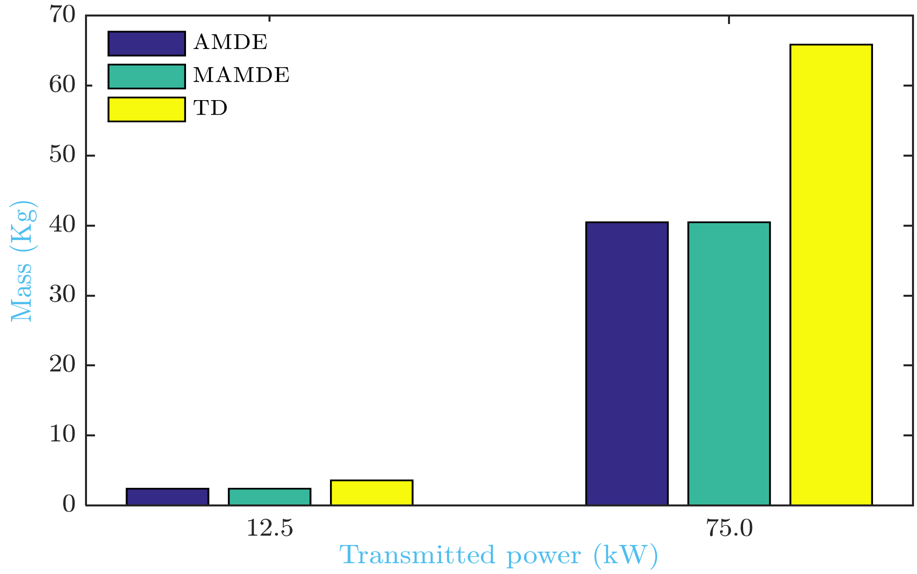


Fig. 5.30 Optimized pinion teeth number (AMDE, MAMDE and TD)

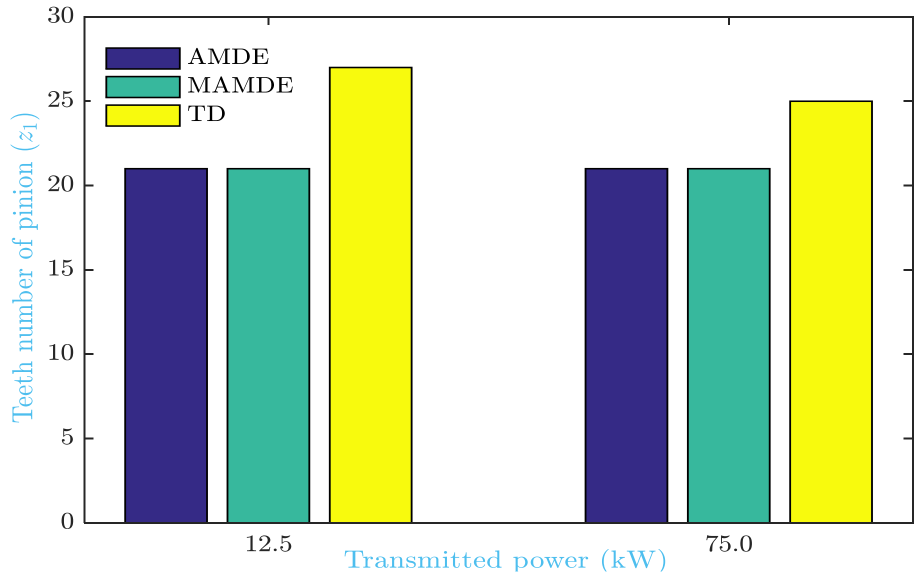
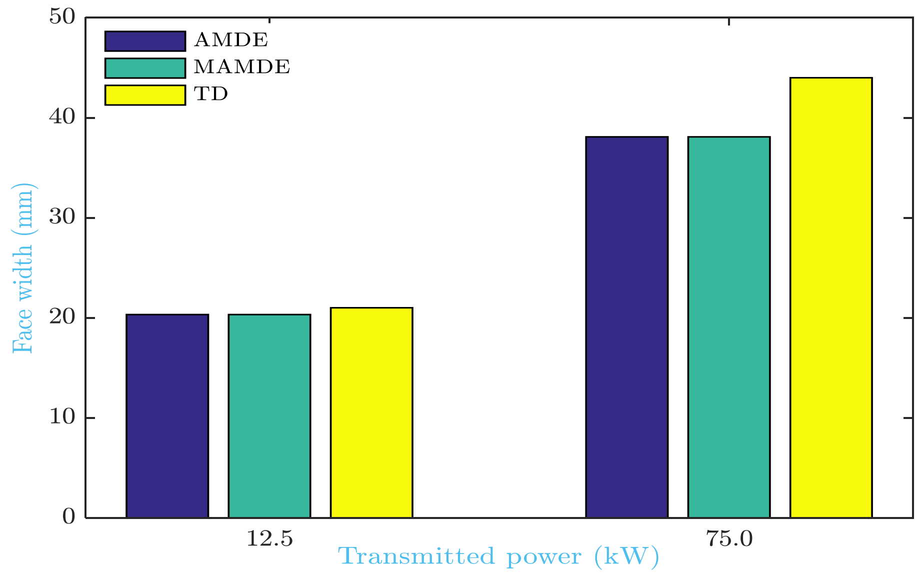


Fig. 5.31 Optimized gear face width (AMDE, MAMDE and TD)



5.4.3.2. Comparison with other algorithms

In the present section, in addition to AMDE and MAMDE ten recent meta-heuristic algorithms are used to optimize both reducers considered in the previous section. Among the methods: Hybrid Multi-PSO (HMPSO) (Wang and Cai, 2009), Dynamic Hybrid Framework (DyHF) (Wang and Cai, 2012a), WCA (Eskandar et al., 2012), Evaporation Rate WCA (ER-WCA) (Sadollah et al., 2015), Improved DE (IDE) (Ho-Huu et al., 2016a), Roulette wheel selection-elitist DE (ReDE) (Ho-Huu et al., 2016b), Combining Multi-objective Optimization DE (CMODE) (Wang and Cai, 2012b), Feasibility Rule with the incorporation of Objective function Information (FROFI) (Wang et al., 2016), Grey Wolf Optimizer (GWO) (Mirjalili et al., 2014) and ABC (Karaboga and Basturk, 2007).

In order to ensure a fair comparison between the used algorithms, the maximum number of function evaluations and the population size are taken the same for all methods, where *NP* = 80, and *FEs* = 150,000. Moreover, we kept the same parameter settings as in the original version for each algorithm, as given in Table 5.8.

Table 5.8 Specific parameter settings of each algorithm

Algorithm	Parameter settings
HMPSO	$F = 0.7$, $Cr = 1$, and $Ns = 8$.
DyHF	$Ns = 10$, $F_1 = 0.7$, $F_2 = 0.5$, $Cr_1 = 1.0$, and $Cr_2 = \begin{cases} 1.0 & if \;\; rand < P_{Cr2} \\ 0.1 & \text{otherwise} \end{cases}$
WCA	$N_{sr} = 8$, $d_{max} = 1\text{E-}03$
ER-WCA	$N_{sr} = 8$, $d_{max} = 1\text{E-}03$
IDE	$F = 0.4 + rand * (1 - 0.4)$, $Cr = 0.7 + rand * (1 - 0.7)$, $\varepsilon_2 = 1.5$ and $\varepsilon_{2max} = 6$
ReDE	$F = 0.4 + rand * (1 - 0.4)$, $Cr = 0.7 + rand * (1 - 0.7)$, $\varepsilon_2 = 1.5$ and $\varepsilon_{2max} = 6$
CMODE	$F = 0.5 + rand * 0.1$, $Cr = 0.9 + rand * 0.05$, $\lambda = 8$, and $k = 22$
FROFI	$F = [0.6, 0.8, 1.0]$ and $Cr = [0.1, 0.2, 1.0]$
ABC	$limit = NP \times n$
GWO	Only the common parameters (*FEs* and *NP*)

where F is the mutation factor, Cr is the crossover rate, Ns is the size of each sub-swarm, ε_2, ε_{2max} are parameters of penalty function, N_{sr} is the number of rivers plus sea, d_{max} is the evaporation condition constant, λ is the number of individuals chosen to take part in DE operations, λ is the generation interval for implementing the feasible solution replacement mechanism, n the number of variables.

To measure the robustness of the compared meta-heuristics in solving the two cases we run each algorithm 20 times. The statistical results including the best, mean, and worst solutions as well as standard deviations. The results obtained by the AMDE, MAMDE and the ten compared methods for the two cases are given in Table 5.9. For more readability, the better solution obtained among the twelve optimizers is in bold.

Considering the first case, it can be seen from Table 5.9 that WCA, ER-WCA, ReDE, GWO and ABC are not able to achieve the best global solution (which is 2.38975 kg) in all runs. Regarding the standard deviation the MAMDE is slightly (1.2294E-03) best than AMDE, FROFI, CMODE, HMPSO, IDE, and DyHF. In addition, among all compared approaches the MAMDE gives the better mean and worst solutions. So, it can be said that the AMDE and MAMDE outperforms the ten compared methods in both solution quality and robustness for the first case study.

Concerning the spur speed reducer (case II), from Table 5.9 it is clear that all compared meta-heuristics are able to optimize this problem in all runs except the WCA, GWO and ABC. However, in terms of the standard deviation, MAMDE is the best one with value of 0.00E+00 followed by AMDE, FROFI and ReDE. Further, our algorithms obtain the best results compared with ten methods in terms of the mean and worst solutions.

From the obtained results for the two reducers, it can be concluded that our developed variants namely the AMDE and MAMDE are very effective in solving the real cases of the mechanical engineering design. In addition, the results confirm that developed algorithms in the present thesis outperform the ten compared methods in both solution quality and robustness.

Table 5.9 Comparison results between AMDE, MAMDE and other algorithms for both case studies

Speed reducer: case I												
	Algorithms											
	HMPSO	DyHF	WCA	ER-WCA	IDE	ReDE	CMODE	FROFI	GWO	ABC	AMDE	MAMDE
Best	2.389752119	2.38975211	2.44952069	2.40849811	2.38975788	2.39111392	2.38975211	2.38975211	2.44949444	2.42581754	2.38975211	2.38975211
Mean	2.395775152	2.39970732	2.77513157	3.13243205	2.39576281	2.40004439	2.39185641	2.39209719	2.55183412	2.45412136	2.39208936	**2.39111287**
Worst	2.408396632	2.44511416	3.58711299	7.61147994	2.40873523	2.40851390	2.40763959	2.40762021	2.74236036	2.50306339	2.40762021	**2.39243653**
SD	7.2536E-03	1.5613E-02	3.0010E-01	1.22933	7.3261E-03	8.3485E-03	3.9011E-03	3.8481E-03	6.8216E-02	2.1676E-02	3.8197E-03	**1.2294E-03**
FEs	150,000	150,000	150,000	150,000	150,000	150,000	150,000	150,000	150,000	150,000	150,000	150,000
Speed reducer: case II												
Best	40.4776837	40.4776837	40.9728626	40.4776893	40.4777300	40.4778360	40.4776837	40.4776837	41.4528921	40.736422	40.4776837	40.4776837
Mean	40.5767185	40.5778870	44.0230698	43.5682527	40.5025834	40.4782841	40.5612462	40.4778017	43.2730513	41.467851	40.4776837	40.4776837
Worst	40.9728578	41.1585849	48.0930720	46.4524613	40.9729997	40.4788256	41.1585849	40.4800447	46.2503386	42.639010	40.4776837	40.4776837
SD	2.0322E-01	2.1324E-01	1.9649E+00	1.9462E+00	1.072E-01	2.8068E-04	2.0703E-01	5.2794E-04	1.2222E+00	4.2819E-01	**1.5971E-14**	**0.00E+00**
FEs	150,000	150,000	150,000	150,000	150,000	150,000	150,000	150,000	150,000	150,000	150,000	150,000

CHAPTER SIX:

Conclusions and future works

CHAPTER SIX

CONCLUSIONS AND FUTURE WORKS

6.1. Conclusions

Nowadays the increasing demand for high strength, efficient, quiet and high precision gear design leads to the use of various optimization methods. However, gear pair design is a highly complex task including different non linear constraints and requires the use of mixed design variables. The main objective of the thesis was the optimization design of the involute cylindrical gears geometry by using an evolutionary algorithm. Among the existing evolutionary techniques the DE algorithm has been chosen. The DE exhibits excellent capability in solving a wide range of optimization problems with different characteristics from several fields and many real-world application problems. In this thesis, two new evolutionary optimization algorithms named AMDE and MAMDE have been presented and investigated based on a self adaptive approach. The modified exterior penalty and the feasibility rules were used to handle the constraints functions. In addition, the variable position index was used in order to treat the discrete and the integer variables.

The effectiveness of the proposed algorithms has been tested by solving twelve problems including six widely used constrained engineering benchmarks (the pressure vessel, tension/compression spring, speed reducer, welded beam, three bar truss, stepped cantilever beam design) as well as six real constrained mechanical design problems (multiple clutch disc brake, coupling with a bolted rim, car side impact, rolling element bearing, step-cone pulley, and hydrodynamic thrust bearing design problem). The simulation results for the tested problems showed that the developed algorithms provide very remarkable results compared to those reported recently in the literature using different optimization methods. Besides, our approaches produced improved results for the coupling with a bolted rim (where the best value of the objective function is improved by 10 %), the rolling element bearing and the step-cone pulley design problem. So, it can be concluded that the AMDE and the MAMDE are very effective algorithms for solving the engineering optimization problems with high quality of solutions and robustness.

It is important to note that the validity of the developed algorithms has been demonstrated by optimizing other mechanical problems such as cam mechanism with offset translating roller follower (Hamza et al., 2018), bearing pivot link (Giraud-Moreau and Lafon, 2002), belleville spring (Rao et al., 2011), reinforced concrete beam design, piston lever, (Gandomi et al., 2013), weight design problem of spur gear (Savsani et al., 2010), size design optimization of truss structures (Ho-Huu et al., 2016), etc. In addition, the capability of AMDE and MAMDE has been evaluated through twenty four constrained test problems collected from the special session on the constrained real parameter optimization of the 2006 IEEE Congress on Evolutionary Computation (CEC 2006) (Liang et al, 2006), and twenty three mathematical functions with different characteristics, i.e. uni-modal, separable and multimodal (Mirjalili et al., 2014). Again, our algorithms can achieve the optimum global solution for all problems, with a good robustness. These results are the subjects of numerous publications in international journals in the near future.

The proposed methods have been applied to solve the problem of the optimal geometry of cylindrical gears. In the first part, a constrained optimization procedure by DE/rand2/bin version has been developed with respect to balancing specific sliding coefficients of involute cylindrical spur and helical gears. A mathematical model of variables was established for all cases of teeth correction. The objective function was defined with purpose to balance specific sliding for the pinion and the wheel at extremes of contact path. Simulation results based on the analyzed gear examples as well as comparisons with other methods demonstrated the effectiveness, efficiency, and robustness of the proposed method. The results obtained by DE lead to a perfect balancing of the specific sliding coefficients.

As second application, the developed methods were applied for solving the optimal tooth profile of a specific cylindrical spur and helical gears. The optimization purposes were to compromise between the maximum bending stresses and the specific sliding. The mathematical model of the maximum bending stresses was developed using a FEA calculations. For the specific spur gear studied in this work, a significant improvement in balancing specific sliding coefficients and maximum bending stresses were found. Moreover, the obtained results give a base for developing complicated mathematical models for the maximum stresses that taking into account all other involved aspects for solving the optimal tooth profile of cylindrical spur and helical gears. For this reason, more advanced methods such as meta-molding and response surface will be used in future works. In addition, a wide range of gear geometries will be considered to guarantee a precise mathematical model.

Finally, in a third step, the developed algorithms have been applied in order to solve the problem of designing a single stage cylindrical speed reducer. The mathematical formulation of the problem takes into account all the standards recommendations specified for complete gear design. For the case study, two speed reducers were considered. Moreover, the optimal results of the developed algorithms were compared with those provided by traditional design method. The obtained results showed a reduction in gearing masses by 33.19 % for the first case and 38.55 % for the second case compared with the results given by traditional method.

6.2. Future works

For future research directions, firstly the MADE and MAMDE will be applied to design various machinery elements and other complex mechanisms such as robot gripper mechanism, cam mechanisms, planetary gear train, straight bevel gears, and complete optimal design of gear train including the shafts, bearings, sub assembly, as well as housing. Secondly, the using of these algorithms will be extended to deal with real applications in automotive industry, manufacturing processes (milling, tuning, etc) and optical elements design. An experimental study to test and to validate the performance of the algorithms is also recommended.

To more speed up the convergence of the proposed algorithms and to improve their local search capability other mechanisms will be used. For the constraints handling more advanced methods will be adopted and employed. In addition, the AMDE and MAMDE can be extended to deal with multi-objective constrained optimization based on the Pareto dominance concept for the selection of generated solutions.

As we know, the deterministic mechanical engineering design optimization does not account for the uncertainties that exist in design variables and mechanical resistance. Due to lack of consideration into uncertainties, the resulting deterministic optimal solution is usually associated with a high chance of failure. For future works, the developed algorithms will be coupled with some advanced reliability theories to account for the uncertainties.

Appendix A: Constrained benchmark mechanical design problems

A.1. Pressure vessel

The problem can be mathematically formulated as follows:

Minimize: $f\left(x\right)=0.6224T_sRL+1.7781T_hR^2+3.1611\,T_s{}^2L+19.84T_s{}^2R$

Subject to:

$g_1(x)=-T_s+0.0193R\le 0,\qquad g_2(x)=-T_h+0.00954R\le 0$

$g_3(x)=-\pi R^2L-\dfrac{\left(4\pi R^3\right)}{3}+1296000\le 0,\qquad g_4(x)=L-240\le 0$

where $0.0625\le T_s\le 6.1875, 0.0625\le T_h\le 6.1875, 10\le R\le 200$ and $10\le L\le 200$.

A.2. Speed Reducer

The problem can be mathematically formulated as follows:

Minimize: $f(x)=0.7854bm^2\left(3.3333z^2+14.9334z-43.0934\right)-1.508b\left(d_1^2+d_2^2\right)+7.4777\left(d_1^3+d_2^3\right)+0.7854\left(l_1d_1^2+l_2d_2^2\right)$

Subject to:

$g_1(x)=\dfrac{27}{b\ m^2z}-1\le 0,\qquad g_2(x)=\dfrac{397.5}{bm^2z^2}-1\le 0$

$g_3(x)=\dfrac{1.93l_1^3}{mzd_1^4}-1\le 0,\qquad g_4(x)=\dfrac{1.93l_2^3}{mzd_2^4}-1\le 0$

$g_5(x)=\dfrac{\sqrt{\left(\dfrac{745l_1}{mz}\right)^2+16.9\times 10^6}}{\left(110d_1^3\right)}-1\le 0,\qquad g_6(x)=\dfrac{\sqrt{\left(\dfrac{745l_2}{mz}\right)^2+157.5\times 10^6}}{\left(85d_2^3\right)}-1\le 0$

$g_7(x)=\dfrac{mz}{40}-1\le 0,\qquad g_8(x)=\dfrac{5m}{b}-1\le 0$

$g_9(x)=\dfrac{b}{12m}-1\le 0,\qquad g_{10}(x)=\dfrac{1.5d_1+1.9}{l_1}-1\le 0$

$g_{11}(x)=\dfrac{1.1d_2+1.9}{l_2}-1\le 0$

where $2.6\le b\le 3.6, 0.7\le \mathrm{m}\le 0.8, 17\le z\le 28, 7.3\le l_1\le 8.3, 7.3\le l_2\le 8.3, 2.9\le d_1\le 3.9$ and $5\le d_2\le 5.5$.

A.3. Welded Beam Design Problem

The problem can be mathematically formulated as follows:

Minimize: $f(x)=1.10471h^2l+0.04811tb\left(14.0+l\right)$

Subject to:

$$g_1(x)=\tau(x)-\tau_{\max}\le 0, \qquad g_2(x)=\sigma(x)-\sigma_{\max}\le 0$$
$$g_3(x)=h-b\le 0, \qquad g_4(x)=0.1047h^2+0.04811tb(14+l)-5\le 0$$
$$g_5(x)=0.125-h\le 0, \qquad g_6(x)=\delta(x)-\delta_{\max}\le 0$$
$$g_7(x)=P-P_c(x)\le 0$$

where $\tau=\sqrt{(\tau')^2+(\tau'')^2+2\tau'\tau''\dfrac{l}{2R}}, \tau'=\dfrac{P}{\sqrt{2}hl}, \tau''=\dfrac{MR}{J}, M=P\left(L+\dfrac{l}{2}\right), R=\sqrt{\dfrac{l^2}{4}+\dfrac{(h+t)^2}{4}}$

$$J=2\left\{\sqrt{2}hl\left[\frac{l^2}{12}+\frac{(h+t)^2}{4}\right]\right\}, \sigma=\frac{6PL}{bt^2}, P_c=\frac{4.013E\sqrt{t^2b^6/36}}{bt^2}\left(1-\frac{t}{2L}\sqrt{\frac{E}{4G}}\right),$$

$P=6000lp, l=14in, E=30\times10^6 psi, G=12\times10^6 psi, \tau_{\max}=13600psi, \sigma_{\max}=30000psi, \delta_{\max}=0.25in,$

$0.1\le h,b\le 2$ and $0.1\le l,t\le 10.$

A.4. Tension/Compression Spring Design Problem

The problem can be mathematically formulated as follows:

Minimize: $f(x)=(N+2)Dd^2$

Subject to:

$$g_1(x)=1-\frac{D^3N}{71785d^4}\le 0, \qquad g_2(x)=\frac{4D^2-dD}{12566(Dd^3-d^4)}+\frac{1}{5.108d^2}\le 0$$

$$g_3(x)=1-\frac{140.45d}{D^2N}\le 0, \qquad g_4(x)=\frac{D+d}{1.5}-1\le 0$$

where $0.05\le d\le 2, 0.25\le D\le 1.3$ and $2\le N\le 15.$

A.5. Stepped cantilever beam design

The mathematical formulation of this problem can be described as follows:

Minimize: $f(x)=100(x_5x_6+x_7x_8+x_9x_{10}+x_1x_2+x_3x_4)$

Subject to:

$$g_1(x)=10.7143-\frac{x_5x_6^2}{10^3}\le 0, \qquad g_2(x)=8.5714-\frac{x_7x_8^2}{10^3}\le 0$$
$$g_3(x)=6.4286-\frac{x_9x_{10}^2}{10^3}\le 0, \qquad g_4(x)=4.2857-\frac{x_1x_2^2}{10^3}\le 0$$
$$g_5(x)=2.1429-\frac{x_3x_4^2}{10^3}\le 0, \qquad g_6(x)=10^4\left(\frac{244}{x_5x_6^3}+\frac{148}{x_7x_8^3}+\frac{76}{x_9x_{10}^3}+\frac{28}{x_1x_2^3}+\frac{4}{x_3x_4^3}\right)-1086\le 0$$

$g_7(x) = x_6 - 20x_5 \le 0,$ $g_8(x) = x_8 - 20x_7 \le 0$

$g_9(x) = x_{10} - 20x_9 \le 0,$ $g_{10}(x) = x_2 - 20x_1 \le 0$

$g_{11}(x) = x_4 - 20x_3 \le 0$

where

$1 \le x_{1,3} \le 5, 30 \le x_{2,4} \le 65, x_5 \in (1, 2, ..., 5), x_6 \in (30, 31, ..., 65), x_{7,9} \in (2.4, 2.6, 2.8, 3.1), x_{8,10} \in (45, 50, 55, 60).$

A.6. Three bar truss

The problem can be mathematically formulated as follows:

Minimize: $f\ (x) = \left(2\sqrt{2A_1} + A_2\right) \times l$

Subject to:

$$g_1(x) = \frac{\sqrt{2}A_1 + A_2}{\sqrt{2}A_1^2 + 2A_1A_2} P - \sigma \le 0, \qquad g_2(x) = \frac{A_2}{\sqrt{2}A_1^2 + 2A_1A_2} P - \sigma \le 0$$

$$g_3(x) = \frac{1}{\sqrt{2}A_2 + A_1} P - \sigma \le 0$$

where: $0 \le A_1 \le 1, 0 \le A_2 \le 1,\ l = 100cm,\ P = 2kN, \sigma = 2kN\ / cm^2.$

Appendix B: Constrained mechanical design problems

B.1. Multiple disc clutch brake

The problem can be mathematically formulated as follows:

Minimize: $f(x) = \pi t \rho \left(r_0^2 - r_i^2\right)(Z+1)$

Subject to:

$g_1(x) = r_0 - r_i - \Delta r \geq 0,\qquad g_2(x) = l_{\max} - (Z+1)\left((t+\delta\right) \geq 0$

$g_3(x) = p_{\max} + p_{rz} \geq 0,\qquad g_4(x) = p_{\max} v_{sr\max} + p_{rz} v_{sr} \geq 0$

$g_5(x) = v_{sr\max} - v_{sr} \geq 0,\qquad g_6(x) = T_{\max} - T \geq 0$

$g_7(x) = M_h - sM_s \geq 0,\qquad g_8(x) = T \geq 0$

where $M_h = \dfrac{2}{3}\mu FZ \dfrac{r_0^3 - r_i^3}{r_0^2 - r_i^2}, p_{rz} = \dfrac{F}{\pi\left(r_0^2 - r_i^2\right)}, v_{sr} = \dfrac{2\pi n\left(r_0^3 - r_i^3\right)}{90\left(r_0^2 - r_i^2\right)}, T = \dfrac{I_z \pi n}{30\left(M_h + M_f\right)},$

Δr = 20 *mm*, t_{max} = 3 *mm*, t_{min}= 1.5 *mm*, l_{max} = 30 *mm*, Z_{max}= 10, v_{max}= 10 *m/s*, μ = 0.5, δ = 0.5 *mm*, M_s = 40 *Nm*, M_f = 3 *Nm*, *input speed* n = 250 *rpm*, p_{max} = 1 *MPa*, I_Z = 55 *kg.m*^{2}, T_{max}= 15 *s*, F_{max} = 1000 *N*, r_{imin} = 55 *mm*, r_{omax} = 110 *mm*, ρ = 7800 *kg/m*^{3}, $r_i \in$ (60, 61, 62,…, 80), $r_0 \in$ (90, 91,…,110), $t \in$ (1, 1.5, 2, 2.5, 3), $F \in$ (600, 610, 620,…, 1000) and $Z \in$ (2, 3, 4, 5, 6, 7, 8, 9).

B.2. Coupling with a bolted rim

The problem can be mathematically formulated as follows:

Minimize: $f(x) = \beta_1\left(\dfrac{N}{N_m}\right) + \beta_2\left(\dfrac{R_B + \phi_4(d) + c}{R_m}\right) + \beta_3\left(\dfrac{M}{M_T}\right)$

Subject to:

$g_1(x) = \dfrac{\alpha M}{NR_B K(d)} - 1 \leq 0,\qquad g_2(x) = 1 - \dfrac{2\pi R_B}{\phi_5(d)N} \leq 0$

$g_3(x) = 1 - \dfrac{R_B}{\phi_4(d)} + R_M \leq 0,\qquad g_4(x) = N - N_{MAXI} \leq 0$

$g_5(x) = R_B - R_{MAXI} \leq 0,\qquad g_6(x) = N_M - N \leq 0$

$g_7(x) = R_M - R_B \leq 0,\qquad g_8(x) = M - M_{MAXI} \leq 0$

$g_9(x) = M_T - M \leq 0,\qquad g_{10}(x) = d - 24 \leq 0$

$g_{11}(x) = 6 - d \leq 0$

where:

$$K(d)=\frac{0.9f_mR_e\pi\left(\phi_1(d)\right)^2}{4\sqrt{1+3(0.16\phi_3(d)f_1/\phi_1(d))^2}},\ M_T=40Nm,\ M_{Max}=1000Nm,\ f_m=0.15,$$

$f_1=0.15,\ \alpha=1.5,\ R_e=627MPa,\ N_M=8,\ N_{Max}=100,\ R_M=50mm, R_{Max}=1000mm,$

$c=5mm, \beta_1=\beta_2=\beta_3=1,\ 6\le d\le 24,\ 8\le N\le 100,\ 50\le R_B\le 100$ and $40\le M\le 100$

Table B.2. Discrete values for bolts

d	$d_e=\phi_1(d)$	$d_2=\phi_2(d)$	$p=\phi_3(d)$	$b_m=\phi_4(d)$	$s_m=\phi_5(d)$
6	5.062	5.350	1.00	7.50	14.50
8	6.827	7.188	1.25	9.50	18.50
10	8.593	9.026	1.50	12.50	23.50
12	10.358	10.863	1.75	13.50	26.50
14	12.124	12.701	2.00	15.50	29.50
16	14.124	14.701	2.00	17.00	32.00
20	17.655	18.376	2.50	21.00	40.00

B.3. Car side impact design

The problem can be mathematically formulated as follows:

Minimize: $f(x)=1.98+4.90x_1+6.67x_2+6.98x_3+4.01x_4+1.78x_5+2.73x_7$

Subject to:

$$g_1(x)=1.16-0.3717x_2x_4-0.00931x_2x_{10}-0.484x_3x_9+0.01343x_6x_{10}\le 1$$

$$g_2(x)=0.261-0.0159x_1x_2-0.188x_1x_8-0.019x_2x_7+0.0144x_3x_5+0.0008757x_5x_{10}+0.080405x_6x_9 +0.00139x_8x_{11}+0.00001575x_{10}x_{11}\le 0.32$$

$$g_3(x)=0.214+0.00817x_5-0.131x_1x_8-0.0704x_1x_9+0.03099x_2x_6-0.018x_2x_7+0.0208x_3x_8 +0.121x_3x_9-0.00364x_5x_6+0.0007715x_5x_{10}-0.0005354x_6x_{10}+0.00121x_8x_{11}\le 0.32$$

$$g_4(x)=0.074-0.061x_2-0.163x_3x_8+0.001232x_3x_{10}-0.166x_7x_9+0.227x_2^2\le 0.32$$

$$g_5(x)=28.98+3.818x_3-4.2x_1x_2+0.0207x_5x_{10}+6.63x_6x_9-7.7x_7x_8+0.32x_9x_{10}\le 32$$

$$g_6(x)=33.86+2.95x_3+0.1792x_{10}-5.057x_1x_2-11.0x_2x_8-0.0215x_5x_{10}-9.98x_7x_8+22.0x_8x_9\le 32$$

$$g_7(x)=46.36-9.9x_2-12.9x_1x_8+0.1107x_3x_{10}\le 32$$

$$g_8(x)=4.72-0.5x_4-0.19x_2x_3-0.0122x_4x_{10}+0.009325x_6x_{10}+0.000191x_{11}^2\le 4$$

$$g_9(x)=10.58-0.674x_1x_2-1.95x_2x_8+0.02054x_3x_{10}-0.0198x_4x_{10}+0.028x_6x_{10}\le 9.9$$

$$g_{10}(x)=16.45-0.489x_3x_7-0.843x_5x_6+0.0432x_9x_{10}-0.0556x_9x_{11}-0.000786x_{11}^2\le 15.7$$

where $0.5\le x_1-x_7\le 1.5, x_8, x_9\in(0.192, 0.345)$ and $-30\le x_{10}, x_{11}\le 30$.

B.4. Rolling element bearing

The problem can be mathematically formulated as follows:

Maximize: $$f(x)=\begin{cases} C_d = f_c Z^{\frac{2}{3}} D_b^{1.8} & \text{if } D_b \le 25.4mm \\ C_d = 3.647 f_c Z^{\frac{2}{3}} D_b^{1.4} & \text{if } D_b > 25.4mm \end{cases}$$

Subject to:

$$g_1(x)=\frac{\phi_0}{2\sin^{-1}\left(\frac{D_b}{D_m}\right)}-Z+1\ge 0, \qquad g_2(x)=2D_b-K_{D\min}(D-d)\ge 0$$

$$g_3(x)=K_{D\max}(D-d)-2D_b\ge 0, \qquad g_4(x)=\zeta B_\omega - D_b \ge 0$$

$$g_5(x)=D_m-0.5(D+d)\ge 0, \qquad g_6(x)=(0.5+e)(D+d)-D_m\ge 0$$

$$g_7(x)=0.5(D-D_m-D_b)-\varepsilon D_b\ge 0, \qquad g_8(x)=f_i-0.515\ge 0$$

$$g_9(x)=f_o-0.515\ge 0$$

where $f_c=37.91\left[1+\left\{1.04\left(\frac{1-\gamma}{1+\gamma}\right)^{1.72}\left(\frac{f_i(2f_o-1)}{f_o(2f_i-1)}\right)^{0.41}\right\}^{10/3}\right]^{-0.3}$, $\gamma=\frac{D_b\cos\alpha}{D_m}$, $f_i=\frac{r_i}{D_b}$, $f_o=\frac{r_o}{D_b}$

$$\phi_0=2\pi-2\cos^{-1}\left(\frac{\{(D-d)/2-3(T/4)\}^2+\{D/2-T/4-D_b\}^2-\{d/2-T/4\}^2}{2\{(D-d)/2-3(T/4)\}\{D/2-T/4-D_b\}}\right)$$

where

$T=D-d-2D_b$, $D=160$, $d=90$, $\beta_\omega=30, 0.5(D+d)\le D_m\le 0.6(D+d), 0.15(D-d)\le D_b\le 0.45(D-d)$, $4\le Z\le 50, 0.515\le f_i\le 0.6, 0.515\le f_o\le 0.6, 0.6\le K_{D\max}\le 0.7, 0.3\le\varepsilon\le 0.4, 0.02\le e\le 0.1, 0.6\le f_i\le 0.85.$

B.5. Step-cone pulley

The problem can be mathematically formulated as follows:

Minimize: $$f(x)=\rho\omega\left[d_1^2\left\{1+\left(\frac{N_1}{N}\right)^2\right\}+d_2^2\left\{1+\left(\frac{N_2}{N}\right)^2\right\}+d_3^2\left\{1+\left(\frac{N_3}{N}\right)^2\right\}+d_4^2\left\{1+\left(\frac{N_4}{N}\right)^2\right\}\right]$$

Subject to:

$$h_1(x)=c_1-c_2=0, \qquad h_2(x)=c_1-c_3=0$$

$$h_3(x)=c_1-c_4=0, \qquad g_{1,2,3,4}(x)=R_i\ge 2$$

$$g_{5,6,7,8}(x)=P_i\ge(0.75*745.6998)$$

where:

- C_i indicates the length of the belt to obtain speed N_i and is given by

$$C_i = \frac{\pi d_i}{2}\left(1+\frac{N_i}{N}\right)+\frac{\left(\frac{N_i}{N}-1\right)^2}{4a}+2a, \quad i=1,\ldots,4$$

- R_i is the tension ratio and is given by

$$R_i = \exp\left[\mu\left\{\pi - 2\sin^{-1}\left\{\left(\frac{N_i}{N}-1\right)\frac{d_i}{2a}\right\}\right\}\right], \quad i=1,\ldots,4$$

- P_i is the power transmitted at each step

$$R_i = st\,\omega\left[1-\exp\left[-\mu\left\{\pi - 2\sin^{-1}\left\{\left(\frac{N_i}{N}-1\right)\frac{d_i}{2a}\right\}\right\}\right]\right]\frac{\pi d_i N_i}{60}, \quad i=1,\ldots,4$$

$$\rho_1 = 7200 kg/m^3,\ a=3m,\ \mu=0.35,\ s=1.75MPa,\ t=8mm.$$

B.6. Hydrostatic Thrust Bearing

The problem can be mathematically formulated as follows:

Minimize: $f(x) = \dfrac{QP_o}{0.7}+E_f$

Subject to:

$g_1(x) = W - W_s \ge 0,\qquad g_2(x) = P_{\max} - P_o \ge 0$

$g_3(x) = \Delta T_{\max} - \Delta T \ge 0,\qquad g_4(x) = h - h_{\min} \ge 0$

$g_5(x) = R - R_o \ge 0,\qquad g_6(x) = 0.001 - \dfrac{\gamma}{gP_o}\left(\dfrac{Q}{2\pi Rh}\right) \ge 0$

$$g_7(x) = 5000 - \frac{W}{\pi\left(R^2 - R_o^2\right)} \ge 0$$

where $W = \dfrac{\pi P_o}{2}\dfrac{\left(R^2 - R_o^2\right)}{\ln\left(\dfrac{R}{R_o}\right)}$, $P_o = \dfrac{6\mu Q}{\pi h^3}\ln\left(\dfrac{R}{R_o}\right)$, $E_f = 9336Q\gamma C\,\Delta T$, $P = \dfrac{\log_{10}\log_{10}\left(8.122e6\mu+0.8\right)-C_1}{n}$,

$$h = \left(\frac{2\pi N}{60}\right)^2\left(\frac{2\pi\mu}{E_f}\right)\left(\frac{R^4}{4}-\frac{R_o^4}{4}\right), \gamma=0.0307,\ C=0.5,\ n=-3.55,\ C_1=10.04,\ W_s=101000, P_{\max}=1000,$$

$$\Delta T_{\max}=50, h_{\min}=0.001, g=386.4, N=750, 1\le R, R_o, Q\le 16, 1\text{e}-6\le\mu\le 16e-6.$$

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Résumé

Cette thèse porte, en particulier, sur l'optimisation de la géométrie des engrenages extérieurs cylindriques droits et hélicoïdaux en développante de cercle en utilisant un algorithme évolutionnaire. Parmi les approches existantes, nous avons choisi l'algorithme à Evolution Différentielle (DE) pour plusieurs raisons: sa simplicité, sa puissance et notamment sa facilité de mise en œuvre. De même, la problématique que nous retenons dans ce travail est liée directement à la résolution déterministe des problèmes d'optimisations mono où/et multi-objectifs des structures mécaniques à variables mixtes. Pour atteindre ces objectifs, de nouvelles versions hybrides de DE sont proposées, à savoir Adaptive Mixed Differential Evolution (AMDE) et AMDE modifié. En outre, les méthodes développées dans ce travail visent à traiter les variables mixtes, éviter le choix manuel des réglages des paramètres de contrôle et d'augmenter la vitesse de convergence du DE.

Dans la première partie de cette thèse, l'efficacité de chaque algorithme proposé est évaluée par l'optimisation de plusieurs problèmes de conception mécanique avec contraintes. Les résultats obtenus montrent que les algorithmes développés fournissent des résultats très remarquables par rapport à ceux récemment rapportés dans la littérature. De plus, de nouvelles solutions optimales pour certains problèmes sont trouvées. Dans la deuxième partie de ce travail, les méthodes proposées sont appliquées pour résoudre le problème de la géométrie optimale des engrenages cylindriques. Nous avons proposé d'abord d'optimiser le problème de la répartition des coefficients de déport entre le pignon et la roue en utilisant la variante DE/rand/2/bin. Dans cette partie, la procédure d'optimisation est développée spécialement pour un équilibrage exact des coefficients de glissement spécifiques aux extrémités de la ligne d'action et tient compte des contraintes de conception des engrenages. Dans la deuxième étape, les algorithmes AMDE et AMDE modifié sont appliqués pour optimiser le profil de dent d'un engrenage droit réel dans une machine de transport. Deux objectifs sont considérés: la résistance à l'usure et les contraintes de flexion maximales des dents d'engrenage. Le modèle mathématique des contraintes de flexion est réalisé à l'aide d'un calcul par la méthode des éléments finis. Pour les deux premières applications, on constate une amélioration significative des coefficients de glissement spécifiques et des contraintes de flexion maximales.

Finalement, dans un troisième temps, les algorithmes développés dans le cadre de cette thèse sont appliqués pour résoudre le problème d'optimisation d'un réducteur à engrenage cylindrique simple étage. La formulation mathématique du problème prend en compte toutes les recommandations de normes spécifiées pour la conception complète des engrenages. Pour l'étude de cas, deux réducteurs de vitesse sont considérés. De plus, les résultats optimaux des algorithmes développés sont comparés à ceux fournis par une méthode de conception traditionnelle. Les résultats obtenus par nos algorithmes ont montré une réduction des masses d'engrenage de 33.19% pour le premier cas et de 38.55% pour le deuxième par rapport aux résultats obtenus par la méthode traditionnelle.

Mots clés: engrenages cylindriques, correction de dentures, déports, algorithme à évolution différentielle (DE), optimisation mixtes, optimisation multi-objectifs, méta-heuristiques.

Abstract

This thesis addresses the optimization design of involute external cylindrical gears geometry by using an evolutionary algorithm. Among the existing approaches, the Differential Evolution (DE) algorithm is chosen for its simplicity, its power and notably eases of implementation. In the same way, this work deals with deterministic solving of single or/and multi-objective constrained engineering design optimization problems of mixed discrete continuous types. To perform these tasks, two new versions of DE algorithm are proposed and investigated, namely Adaptive Mixed Differential Evolution (AMDE) and modified AMDE. Furthermore, the developed methods in this work aim to handle the mixed variables, to avoid the manual choice of control parameter settings and to increase the convergence speed of the DE.

In the first part of the thesis, the performances of the proposed algorithms are numerically tested using several constrained mechanical design problems. The simulation results for the tested problems show that the developed variants provide very remarkable results compared to those reported recently in the literature using various meta-heuristic algorithms. Besides, new optimal solutions for some engineering problems are obtained. In the second part of this work, the proposed methods are applied to solve the problem of the optimal geometry of cylindrical gears. The optimal selection values of profile shift coefficients for cylindrical involute spur and helical gears is performed using the DE/rand2/bin version. In this part, the optimization procedure is developed specifically for an exact balancing of specific sliding coefficient at extremes of contact path and account for gear design constraints. Secondly, the AMDE and the modified AMDE algorithms are applied to optimize the profile tooth of a real spur gear in a large transport machine. Two objectives are considered, the wear resistance and maximum bending stresses of gear teeth. The mathematical model of the maximum bending stresses is developed using the finite element analysis (FEA) calculation. For the two first applications, a significant improvement in balancing specific sliding coefficients and maximum bending stresses are found.

Finally, in the third step, the developed algorithms are applied in order to solve the problem of designing a single stage cylindrical speed reducer. The mathematical formulation of the problem takes into account all the standards recommendations specified for complete gear design. For the case study, two speed reducers are considered. Moreover, the optimal results of the developed algorithms are compared with those provided by traditional design method. The obtained results by our algorithms showed a reduction in gearing masses by 33.19% for the first case and 38.55% for the second case compared with the results given by traditional method.

Keywords: cylindrical gears, tooth modifications, profile shift coefficients, differential evolution (DE) algorithm, mixed discrete-continuous constrained optimization engineering design, multi-objective optimization, meta-heuristic algorithms.