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On recent descent methods for large-scale optimization

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ملخص:

هذه الأطروحة تتناول دراسة نظرية وعددية لبعض خوارزميات التدرج المترافق لحل مسائل الأمثلة غير مقيدة، حيث تكون الدالة غير خطية ولكنها تفاضلية. لدراسة أداء الطرق المتتالية، تم إجراء تجارب عددية على مجموعة من دوال الاختبار.

كلمات مفتاحية : الأمثلة غير المقيدة، الأمثلة غير الخطية، طرق التدرج المترافق، البحث الخطي، التقارب الشامل.

Abstract :

In this study, we present a synthesis of various conjugate gradient methods for solving unconstrained optimization problems, where the objective function is nonlinear but continuously differentiable (and possibly non-convex). To illustrate the performance of these methods, numerical experiments are conducted on a set of standard test functions, along with comparative analysis.

Key-words: Unconstrained optimization, Nonlinear optimization, Conjugate gradient methods, Line search, Global convergence.

Résumé :

Dans cette étude, nous présentons une synthèse de différentes méthodes de gradient conjugué pour la résolution de problèmes d'optimisation sans contraintes, où la fonction objectif est non linéaire mais continûment différentiable (et éventuellement non convexe). Afin d'illustrer les performances de ces méthodes, des expériences numériques sont menées sur un ensemble de fonctions de test standard, accompagnées d'une analyse comparative.

Mots-clés : Optimisation sans contraintes, Optimisation non linéaire, Méthodes de gradient conjugué, Recherche linéaire, Convergence globale.

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