



Setif 1 University Ferhat Abbas
Faculty of Sciences
Department of Mathematics



جامعة سطيف 1 فرحات عباس
كلية العلوم
قسم الرياضيات

Master's Thesis in Mathematics

Option : Optimization and Control

Theme :

**On a variant of primal-dual interior point method
for linear optimization**

Presented by

MIHOUBI Malek

BOUCEKKINE Oumaima

Supervisor: **Pr. BENTERKI Djamel**

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Ms. CHAGHOUB Soraya

MCB. Setif 1 University Ferhat Abbas

President

Mr. BENTERKI Djamel

Prof. Setif 1 University Ferhat Abbas

Supervisor

Ms. BOURAS Ikram

MCB. Setif 1 University Ferhat Abbas

Examiner

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ملخص :

في هذه المذكرة، نهتم بحل مسألة برمجة خطية باستخدام طريقة النقط الداخلية الأولية-الثنائية. استلهاً من عمل زانغ وكسو (2011)، نقترح اتجاه انحدار جديد من خلال تحويل معدل لشرط التكامل في النظام الذي يحدد المسار المركزي. من أجل ذلك قمنا بإجراء دراسة نظرية وعددية كاملة.

كلمات مفتاحية: البرمجة الخطية، طريقة النقط الداخلية، اتجاه انحداري، تقارب خوارزمي.

Abstract:

In this manuscript, we are interested in solving a linear programming problem using a primal-dual interior point method. Inspired by the work of Zhang and Xu (2011), we propose a new descent direction through a modified transformation of the complementarity condition in the system that defines the central path. A comprehensive theoretical and numerical study has been carried out to achieve our objective.

Keywords: Linear programming, Interior point method, Descent direction, Algorithmic complexity.

Résumé :

Dans ce mémoire, on s'intéresse à la résolution d'un problème de programmation linéaire en utilisant une méthode de points intérieurs primale-duale. En s'inspirant du travail de Zhang et Xu (2011), on propose une nouvelle direction de descente à travers d'une transformation modifiée de la condition de complémentarité dans le système qui définit le chemin central. Une étude théorique et numérique complète a été menée pour atteindre notre objectif.

Mots clés : Programmation linéaire, Méthode de points intérieurs, Direction de descente, Complexité algorithmique.